

Towards a Platonic Theory of Wholes and Parts

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Abstract of Thesis

The aim of this thesis is to introduce and elaborate a new conception of the relation between wholes and parts. Wholes, I propose, can be conceived of as 'Unities', in contrast to their currently familiar conception as 'sums'. Following a clue given in the distinction which Plato draws in the *Theaetetus* (203c-205e) between two conceptions of a complex entity, I argue that a similar distinction can be coherently developed in modern terms.

Part I is preoccupied with general conceptual and historical background. Some theoretical constraints on any theory of wholes and parts are challenged and found to be merely apparent.

In Part II the conception of wholes as sums is presented, and it is extensively argued that modern discussions of wholes generally presuppose this conception. This presupposition is shared not only by authors who subscribe to the 'classical' mereological theories of Lesniewski, and Goodman, but also by theorists of holistic sympathies (making use, for example, of the notion of an organic whole, or of a Gestalt) who rely on 'neoclassical' theories. It is urged that this conception suffers from serious, fundamental difficulties and drawbacks.

In Part III the conception of wholes as Unities is introduced. A theory of Unities is laid down in systematic, formal detail, and the points of divergence from presuppositions of traditional theories are discussed. It is shown how in conceiving of concrete entities (of certain types) as Unities one is free from many difficulties which beset their conception as sums. Finally, it is shown how the theory of Unities provides a powerful tool for resolving some central metaphysical puzzles concerning concrete entities, especially puzzles associated with preservation of identity in the face of loss or gain of parts.

Declaration of Originality

This thesis has been composed by me, and the work which it represents is my own.

For Rachel and Moshe Meirav

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Introduction

To study the notion of a whole is to study a wide ranging constraint on one's view of the world. Given the assumption that an entity is made of certain parts, one's notion of a whole places constraints on the character of that entity. For example, if one assumes that whenever parts make up a whole, that whole cannot exist unless each of those parts exists, then one is abandoning the idea that there are entities in the world which survive the loss of some of their parts. Different understandings of the notion of a whole carry with them, therefore, correspondingly different constraints on the way we conceive of entities with which our world is populated.

It transpires that the confrontation between notions of a whole may be a confrontation between different, indeed radically different, views of the world. The forging of a new notion may well have the consequence of removing a constraint that had previously been taken for granted.

In setting out different notions of a whole, authors have often attempted to bring out the essential features of a whole which complies with their notion, by contrasting it with a whole which lacks those features. Several general approaches are familiar in drawing such contrasts. According to one approach, the notion of a whole whose parts bear particular types of relations to one another is contrasted with the notion of a whole whose parts are not similarly related. For example, sometimes a contrast is drawn between a whole whose parts have a certain kind of causal relations to one another, and a whole whose parts do not. Or, to mention another example of this general approach, sometimes a contrast is drawn between a whole whose parts depend on one another for their existence, and a whole whose parts are independent in this respect.

Other familiar approaches focus on different aspects of wholes, instead of the relations between the parts: according to one approach the notion of a whole which

has properties (for example, so called 'emergent' properties) that are not related in obvious ways to the properties of the parts, on the one hand, is contrasted with the notion of a whole which does not possess such properties, on the other hand. Another suggestion emphasizes a contrast regarding conditions for the survival of wholes - the notion of a whole which may survive the loss of some of its parts is contrasted with the notion of a whole which cannot survive under such conditions.

The present work is devoted to the elaboration of a notion of a whole which cannot be accounted for in terms of any of these familiar contrasts. I call wholes of this type 'Unities', and consider it appropriate to describe the theory of Unities as a 'Platonic' theory of wholes and parts.

By saying that the theory of Unities is a *Platonic* theory I wish to allude to the source of the reflections which ultimately led to the account presented in this work. The work began as an investigation of what I took to be a theme in Plato's thinking about parts and wholes, a theme which expressed itself most notably in a well known passage towards the end of his dialogue *Theaetetus* (203c4-205e8). Ironically, I found that the attempt to make sense of the passage required a preliminary conceptual clarification of such a scope as to leave no space for treating the passage itself. For I was unsatisfied by the picture which resulted from reading any of a range of familiar modern views on parts and wholes into Plato's argument.

As a consequence, the investigation to be found in the following pages, although deeply inspired by Plato's thought, does not include an interpretation of any passage of Plato's. When I describe the theory as 'Platonic', I do *not* mean to imply that this, in my view, is Plato's theory of wholes and parts. My views are importantly related to what I take Plato's views to be. But to spell the relationship out in detail would require an extended discussion which cannot find its place in the work that follows. In particular, I do not pretend to be offering here a reading of the crucial passage of *Theaetetus* just mentioned. I trust that such a reading will be greatly helped by the present investigation. It will not be undertaken here, however.

The epithet 'Platonic', however, should not be viewed merely as a symbolic token of personal indebtedness. Although the account of parts and wholes found in

Theaetetus is profoundly puzzling and difficult to interpret,¹ the following three uncontroversial points can be made in its regard. (1) Much like the more familiar approaches mentioned above, Plato's account distinguishes between two fundamentally different notions of an entity which is made up of many (i.e. more than one) entities. However, (2) the account does *not* attempt to explicate this distinction in any of the three ways mentioned above. Furthermore, again by contrast with the more familiar approaches, (3) his account lays an emphasis on the assumption that according to the first notion, an entity is *identical* to the entities which make it up, whereas according to the second notion it is distinct from them. In each of these respects, my account parallels that of Plato.

Indeed, this work can be seen, at least in part, as a response to the challenge - which struck me intuitively as a worthwhile and promising challenge - of arriving at a coherent account of wholes which shares these basic points with Plato's account. The theory of Unities, presented in Part III, is a theory which accommodates two distinct notions of a whole, where the contrast between the notions is one in which relations between parts, emergence, or conditions of survival play no role. Furthermore, I argue that a whole which corresponds to the first notion is identical to the parts which make it up, whereas a whole which corresponds to the second notion is not identical to its parts.

It might be asked, why we should desire to have such an account. One answer is, simply, that it is intrinsically interesting that such an account should be available. If the comment with which I opened this Introduction is accepted, the exploration of an unfamiliar notion of a whole is at the same time an exploration of the possibility of adopting unfamiliar assumptions at the roots of our view of the world.

A more specific answer concerns the realization that, as I shall argue, familiar accounts of wholes and parts, in general, meet with serious problems, when one attempts to apply them as part of a theory of concrete entities that endure in time, that is, of continuants. If my analysis is correct, some of these problems may be

¹ For some of the difficulties raised by the passage, see in particular Burnyeat 1990, 191-209, McDowell 1973, 241-6, Miller 1992, 93-95, Scaltsas 1990, 583-4. Further items included in the bibliography, which illuminate various respects of the context in which the passage appears, are the following: Bostock 1988, Cornford 1988, Fine 1979, Gallop 1963, Morrow 1970, Ryle 1960, Ryle 1990, Sayre 1969.

treated in a surprisingly straightforward and satisfying way if one conceives of continuants as Unities. Indeed, some of the most challenging puzzles of the metaphysics of continuants receive new, and what seem to be very promising solutions.

One of the more interesting results of the theory of Unities, concerns the account it offers for the (controversially) alleged possibility that two entities should be distinct from one another, notwithstanding the fact that they are superposed with one another, that is, that each of them is made up of precisely the same entities. Familiar views of parts and wholes seem to be able to explain this possibility only by reference to differences between the properties the two entities have at other times, or in other possible worlds. The theory of Unities, by contrast, makes it possible to explain this without reference to such temporally or modally remote properties.

In the first place, this enables the theory of Unities to avoid certain problems which, as I argue, face the more familiar views in connection with accounting for superposition of continuants. In addition, however, it means that the theory of Unities is much better suited than the familiar theories to offering an account of analogues of the phenomenon of superposition in connection with entities that do *not* have different properties at different times, or in different possible worlds. This aspect of the theory of Unities indicates that it may hold much interest in the context of the study of complexes of universals, or of linguistic complexes (e.g. propositions).

Inevitable limitations of scope have made it necessary to confine the discussion in this work to the theory of continuants. Indeed, I shall be treating the theory of Unities as a mereological theory of continuants. I think it will be clear to the reader, however, that a theory based on very similar principles can be formulated in such a way as to be applicable to entities that are not continuants. The view of an interesting field of application of this sort, which lies somewhat beyond the confines of the thesis itself, gives us additional reason to be interested in looking into the conception of wholes which the theory proposes.

In broad outline, the strategy of the work is as follows. In Part I basic conceptual issues prior to the theoretical account of wholes are discussed. I attempt

to introduce the notion of a whole from a perspective which precedes, and is independent of, those of the theories of wholes which I later discuss. This will help us to consider those theories more open-mindedly, for the notion of a whole will not have been introduced in a way which is loaded with assumptions which are particular to one of the theories. Part I, therefore, sets in place the notion for which the theories discussed in Parts II and III aim to provide a theoretical account. Part II concerns the presently available theoretical account. Part III concerns the proposed alternative in the shape of the theory of Unities.

The available account is characterised in Part II as the conception of wholes as *sums*. Wholes are understood as sums, in this sense, not only by theorists who uphold the principles of classical mereology. In fact, the conception is sufficiently adaptable to admit such features of wholes as superposition, flux of parts, and conditional composition. Indeed, I argue that even traditional “higher-grade” notions of a whole, such as the notion of an organic whole, or that of a Gestalt - at least in as far as clear accounts of these notions have been proposed - find a comfortable place within the framework of wholes as sums.

However, as I explain, the notion of a sum has clearly specifiable limitations. Notwithstanding its impressive adaptability, it encounters serious difficulties. Interestingly, some of these problems are directly traceable to those limitations.

The alternative proposed account - according to which (some) wholes are Unities - is developed in detail in Part III. The notion of a Unity is based on the rejection of a certain principle which is implicit in the conception of wholes as sums, namely, the assumption that wholes are what I call ‘coextensively determined’. To make this assumption is to assume that if the *xs* make up a whole, and the *ys* make up the same whole, then the *xs* must be ‘coextensive’ with the *ys*, that is, roughly, they extend (if taken together) over precisely the same region of space. The theory of Unities rejects this assumption. I argue that one may reject this assumption without conflicting with broadly agreed upon assumptions regarding the relation *is a part of*. This contributes to the plausibility of the proposal.

Nevertheless, to reject this assumption involves a multifaceted change in one’s conception of wholes, a change which I try to bring out and illustrate in various ways. One of these is to draw parallels between what I say about the relation between wholes and parts that make it up, and what is clearly the case in

connection with some non-mereological relations. The relation between a Unity and entities that make it up comes to be seen not as a suspicious *sui generis* relation, but rather as a relation that is interestingly similar to some familiar non-mereological relations.

In addition to pointing out such parallels, I show how the rejection of the above-mentioned principle removes the specific difficulties which were raised in connection with the conception of wholes as sums. I conclude, finally, by showing in detail how the resulting theory of Unities fares in comparison with theories based on the conception of wholes as sums, in dealing with one of the more famous puzzles associated with flux of parts.

A minor technical comment: the work is divided up hierarchically into Parts, chapters, Sections, Subsections and Items. Occasionally I will say such things as “in the next Part ...”, “in the last Subsection”, “in the first two Sections of ...” “throughout the present chapter”, “before turning to the next Item”, etc.. The use of upper case letters here (except in the case of ‘chapter’) is simply meant to indicate that I am referring to divisions of this work.

I Wholes

Chapter 1

Concrete Comprising Entities

Section 1.1

Preliminaries

1.1.1 *Comprising Entities in General*

It is a familiar and pervasive feature of thought and speech, that often when we are able to refer to many distinct entities, x_1, x_2, \dots, x_n , we also seem to be able to refer to an entity y which in some loose sense we take to include or contain x_1, x_2, \dots, x_n . Let us say that in such cases y *comprises* x_1, x_2, \dots, x_n , and so describe y as a *comprising* entity. A tree, we may then say, comprises its cells. The set, whose members are the tree's cells, may also be said to comprise those cells. We usually consider the relation between the cells and the tree as the relation of *being a part of*. The relation between the cells and the set of cells we normally take to be a different relation, that of *being a member of*. However, these relations have some general features in common which seem to justify taking them to be species of one and the same generic relation.

In the first place, while the cells are *many* entities, the tree is *one* entity, and similarly the set of cells is *one* entity: thus both relations are relations between many

entities and one entity. Secondly, there is a sense in which the existence of both the tree and the set of cells depends necessarily on the existence of the cells. Suppose the cells are parts of the tree at time t , and consider the set S of all possible worlds in which entities exist only momentarily, at time t . Consider a subset of S , members of which are worlds in which the cells do not exist (nor any entity whose existence necessarily depends on the existence of the cells), but which are otherwise indiscernible from the actual world at t . Neither the tree, nor the set of cells, will exist in these latter possible worlds. Moreover, it seems clear that the necessity involved here is not relative to the laws of nature - that it is a metaphysical necessity rather than a nomological one.

Thirdly, with regard both to the tree and the set of cells, it seems to be an appropriate use of metaphor to say that the cells *belong* to them: the cells belong to the tree, and the cells belong to the set (though admittedly 'belong' does not have precisely the same sense in the two cases). Finally, it is at least *prima facie* plausible loosely to describe speaking of either the tree or the set by saying, that in either case we speak of the cells "as one entity".

We say, therefore, that both the tree and the set are *comprising entities*, and that the cells are comprised by both the tree and the set. By contrast, it is clear that the relations between David and his many friends, and between David and his many books, are not relations between comprising and comprised entities, although both are relations between one entity and many entities, and the verb 'to belong' is used (again, metaphorically) to describe the relation between the many entities and the one entity. For the second and fourth feature are missing in these cases.

Ultimately, however, the decision to speak about a generic comprising relation need not be treated as more than a heuristic device, a way of focusing on several very general problems without getting bogged down, at this early stage of the discussion, in the numerous terminological distinctions which abound in this field. With this in mind, it seems best to avoid attempting a precise characterisation of the comprising relation, and instead to have before us typical specific examples.

Consider therefore the following table, in which expressions designating many comprised entities are listed in the left-hand column, and expressions designating corresponding comprising entities are listed in the right-hand column:

<u>COMPRISED ENTITIES</u>	<u>COMPRISING ENTITY</u>
stones	wall
molecules	stone
molecules	tree
cells	organism
cells	collection of cells
pebbles	collection of pebbles
pebbles	class of pebbles
pebbles	set of pebbles
birds	flock
players	team
grains of sand	heap
buildings	group of buildings
stars	star-cluster
meteorites	collection of meteorites

The expressions listed in the left hand column are in the plural, syntactically speaking: they combine with verbs in the plural to form grammatical sentences. Thus on syntactical evidence each of them is apparently used to refer to many entities. The expressions listed in the right hand column are syntactically either singular or indeterminate (with respect to number). 'The organism' combines with verbs in the singular to form grammatical sentences, while 'the team' combines either with verbs in the singular or with verbs in the plural to form grammatical sentences.

On syntactical evidence, again, each of the singular expressions is apparently used to refer to one entity, as does each of the indeterminate expressions in contexts where it combines with verbs in the singular. I shall assume in what follows, unless explicitly stated otherwise, that indeterminate expressions are found in such

contexts, and thus I shall treat them as syntactically singular. I will take it, then, that each of the expressions in the left hand column is apparently used to refer to many entities, whereas each of the expressions in the right hand column is apparently used to refer to one entity.

One point to emphasize about my use of 'comprises': I say that the organism comprises the cells; I do not say that the organism comprises *each* of the cells. 'comprises' expresses a certain relation which holds between the organism, on the one hand, and the cells taken together, on the other hand.

1.1.2 *Controversial and Uncontroversial Comprising Entities*

Regarding some of the examples in the table, the existence of a comprising entity is not controversial, or at least no more controversial than the existence of the comprised entities. It is normally assumed, for instance, that there exists an entity to which 'organism' is appropriately used to refer, no less than it is assumed that there exist entities referred to by means of the expression 'cells'. What is controversial here is how we are to conceive of the relations between the comprising and comprised entities. Pre-theoretically, it is a relation between a whole and many parts which compose it. Various theoretical accounts have been offered for this relation, however, and each has its own difficulties. We shall be looking more closely at the theory of wholes and parts in later chapters.

Regarding some of the other examples in the table, however, the *existence* of a comprising entity is much closer to the foreground of controversy. Common sense does not seem to have as confident a position as regards the existence of entities to which expressions such as 'team of players', 'flock of birds', 'collection of pebbles' or 'set of pebbles' might be used to refer, as it did regarding the existence of a tree, or of organisms in general.¹

¹ Doubts of this sort arise in particular in connection with many terms which are used in relatively narrow technical senses by various authors in their discussion of comprising entities. Perhaps most familiar in this connection are doubts associated with the terms 'set' or 'class', as used in set theory. Russell and Whitehead 1910 argue that classes are logical fictions; Goodman criticises the notion of a class e.g. in Goodman 1956, 199-203. As regards

Suppose, for example, that 'collection of pebbles' is used in a context in which more than one pebble is supposed to be a member of the collection. It might be suggested that the sentence 'the collection of pebbles is being moved' (for example) is identical in meaning to 'the pebbles are being moved'; and that therefore it is inappropriate to acknowledge in this context the existence of any entities to which the latter sentence does not commit us. But the latter sentence commits us only to the existence of the many pebbles, and the collection of pebbles, if there were such a thing, could not be identical to any of the pebbles. Alternatively, it might be suggested - if it is denied that the two sentences are identical in meaning - that the notion of a collection is obscure, and that the existence of an entity corresponding to such an obscure notion must be doubted.²

In the case of a doubtful comprising entity the question whether the entity exists is closely bound up with the question how such an entity is to be conceived of. For example, if one takes it that the notion of a collection must be construed as the notion of a certain type of abstract entity,³ then if one denies the existence of abstract entities one will also deny the existence of a collection. One might then say

other allged comprising entities, see, e.g., Russell's sceptical remark regarding items designated by 'heap' or 'conglomeration' in Russell 1919, 183.

Among technical terms used in discussion of comprising entities one finds in addition to 'whole', 'sum', 'class', 'set' and 'collection' (of which I make use in what follows), terms such as:

'fusion' (see e.g. Lewis 1991, 72-4)

'manifold', 'domain', 'aggregation' (see e.g. Frege 1895, 86-7)

'heap', 'aggregate' (these two terms seem to be treated as synonymous by Armstrong - see 1978, 29ff.; his comments p. 31 suggest that he takes these terms in Goodman's sense of 'sum' (or 'sum-individual' - see 1956, 201); Quine uses 'heap' in this sense in 1953, 114; for earlier use of 'aggregate', see Russell 1903, 138-140; both terms are common in commentary on Aristotle, as translations of the Greek '*soros*'; Miller (1978, 112) argues that in some contexts, though not all, Aristotle uses '*soros*' in a technical sense; the translation 'aggregate' seems to emphasize the technical sense - see particularly the account of the notion in Scaltsas 1994b, 66, and 1994a, 111. For further examples of use of 'heap' and/or 'aggregate' in commentary on Aristotle, see e.g. Furth 1988, 294 (index item); Haslanger 1994, 140; Halper 1989, 139, 151; Bogaard 1979, 12)

'conglomeration' (see e.g. Russell 1919, 183)

'group' (see e.g. McTaggart 1921, Ch. XV; Sprigge 1970, 112ff.)

'totality' (see e.g. Küng 1967, 105-8)

Although I take the notions associated with each of these terms to be notions of comprising entities, I do not attempt in the present work to settle their precise relations to notions that occur in my discussion.

² See Max Black's claims regarding the obscurity of the notion of a set (Black 1971); see also his comments on the notions of a collection and an assembly, p. 619-22.

³ For comment on the contrast between concrete and abstract entities, see 1.1.3.1 below.

that notwithstanding the linguistic evidence according to which reference is apparently made to a comprising entity, analysis reveals that this reference is only apparent, and no comprising entity exists which would correspond to the expression 'the collection' in the intended contexts.

Furthermore, if for example one speaks of a collection of cells, one is likely to be led much more directly to a consideration of the relation between the ostensible comprising entity and the individuals it comprises, than one would if one spoke of an organism. For one is more likely to be faced immediately with clarificatory questions such as: When you speak about the collection of cells, do you mean simply to speak about the many cells, or do you mean to distinguish talk about the collection from talk about the many cells? Of course such questions can be asked in connection with the organism no less than they can with regard to the collection of cells. But in the case of the collection they seem, as it were, closer to the surface, and answers would seem to be more urgent for any discussion of the ostensible entity at hand to proceed with clarity.

Taking these points into account, one is less surprised to realize that the principal current theories which attempt to account for the relations between *uncontroversial* comprising entities and the individuals they comprise (e.g. the organism and the many cells) are descendants of 19th century theories which were principally concerned with *controversial* comprising entities (e.g. classes, sets, manifolds). If one wishes to examine fundamental features of current theories of wholes and parts, therefore, it seems reasonable to begin by concentrating on conceptions of ontologically more controversial or doubtful comprising entities. This is an important reason for the emphasis laid in the present Part on the discussion of the latter type of comprising entities.

Ultimately, however, the present work's chief concern is to examine the conception of *uncontroversial* comprising entities. I take the following to be among uncontroversial comprising entities: organisms (such as trees, whales, viruses), more or less clearly delimited chunks of matter (such as rocks, mountains, lakes, planets), artefacts (such as houses, guitars), and various parts of any of these (such as cells, kidneys, molecules). These are all concrete entities, and they are all, at least *prima facie*, individuals. It is also noteworthy that all are designated by count-nouns. However, not anything designated by a count-noun need be an individual, *prima*

facie (it is not clear, for instance, whether galaxies are counted, *prima facie*, as individuals; and yet 'galaxy' is a count-noun). Our investigation does not require a precise delineation of the range of entities with which we shall be concerned.^(in Parts II and III) However, since it has been described as the range of concrete entities which are *prima facie* individuals, a brief explanation is called for of the way I understand the notions of concreteness and of being an individual.

1.1.3 *Entities that are prima facie Individuals*

1.1.3.1 *Concreteness*

It is by no means an easy matter to define concreteness satisfactorily.⁴ For our purposes, however, it is sufficient to take the claim that an entity is concrete as equivalent to the claim that (1) it has a more or less definite three-dimensional spatial location, and (2) it is a continuant rather than an occurrent (at least *prima facie*),⁵ and (3) it moves or is at rest (either in the sense that it might move or be at

⁴ For a discussion of difficulties in defining concreteness, see Hale 1987, ch.3.

⁵ The distinction between continuants and occurrents can be intuitively described as the distinction between entities which *endure* through time and entities which *extend* through time, respectively. (An entity is *prima facie* a continuant if *prima facie* it is an entity which endures, and does not extend, through time). Following C. D. Broad (1933), Simons explains this distinction in terms of the contrast between entities which do not have temporal parts, and entities which do. A precise account of the notion of a temporal part is quite an involved matter (see Simons 1987, 132). Mellor (1995, 123) is able explain the distinction between continuants and occurrents in an apparently simpler way, by assuming that something can be a genuine (proper) *part* of something else only if its existence is logically independent of the existence of the latter.

An account which neither makes Mellor's assumption, nor makes use of the notion of a temporal part, can be offered as follows:

Assume the predicate '*x* exists during *I*' is (primitively) understood, wherever *x* is an entity and *I* is a temporal interval. Then '*x* exists at *t*' can be defined as follows:

x exists at *t* =_{def.}
x exists during an infinitesimal interval to which *t* belongs.

Then 'continuant' and 'occurrent' (or more precisely, '*x* is a continuant' and '*x* is an occurrent') can be defined as follows:

x is a continuant =_{def.}
x exists during interval *I* iff

rest relative to other entities, or in the sense that its parts might move or be at rest relative to one another) and (4) it has mass (or weight). Thus properties, classes, mathematical objects, states of affairs, events, images, sounds, smells, and various "non-material" items such as minds, ghosts or angels count as non-concrete. This may well be thought to exclude too much from the category of concrete entities, and thus it is not plausible to treat this criterion as a definition of 'concrete entity'. It does, however, serve well to mark out a type of concrete entity on which our interest will be focused.

1.1.3.2 *Being an Individual (A): non-attributability*

The notion of an individual is generally understood as the notion of an entity which has a feature which might be described as 'non-attributability'. That is to say, individuals are entities which have properties and are related by relations, but they are not themselves properties of, or relations between, other entities. Thus, for example, John is an entity which is not a property of (or a relation between) other entities, and so (according to this criterion) John is an individual. While *weighs 500 pounds* is not an individual, for it is an entity which is a property of other entities.

Sometimes this contrast between entities which are not, and entities which are, properties or relations is understood in terms of the contrast between membered and unmembered entities: that is, between entities which have members, and entities which do not.⁶ In these terms, John is an individual, because John is an entity which does not have members. While the class of 500 pound entities is not an

for all t , such that t belongs to I , x exists at t .

x is an occurrent =_{def.}

x exists during interval I iff

(1) for all t , such that t belongs to I , for some y ,

y is a part of x and y exists at t .

and

(2) for all t and t' , such that both t and t' belong to I , but $t \neq t'$,

and for all y and for all z ,

if y is a part of x and z is a part of x , and y exists at t and z exists at t' ,
then $y \neq z$.

⁶ See e.g. Lewis 1991, 1-10.

individual, because it is an entity which does have members. The motorcycle parked on the pavement there is one such member, assuming that it weighs 500 pounds.

The feature of non-attributability is clearly possessed by all concrete entities. If an entity moves, for example, it is not a property of anything, nor a relation between things; for properties and relations do not move.

1.1.3.3 *Being an Individual (B): unity*

Entities that are *prima facie* concrete individuals have, in addition to the features of concreteness and non-attributability, a further distinctive feature which can be described by means of the term 'unity'.⁷ To say that a comprising entity is unified, in this sense, is to say that the entities it comprises are closely held together - a rather vague notion which has, nevertheless, quite a strong intuitive appeal. As an example of an entity which is *prima facie* a concrete individual, consider a whale, which comprises its many bodily organs. Compare the whale with a *pair* of whales, an entity (indeed, if such an entity exists at all) which is not, *prima facie*, an individual; this entity may be assumed to comprise the many bodily organs of *two* whales. In the former case, the comprised entities seem to be closely held together in a way that they do not in the latter case. One expression of the contrast between these examples is that the organs of the single whale meet conditions of spatial continuity, which those comprised by the pair of whales do not. Another expression of the contrast is that the well-being of one of the organs of the single whale may influence the well-being of all the other comprised organs; the organs comprised by the pair of whales are not similarly interdependent.

⁷ The use of 'unity' here, as expressing a *feature* of entities, should not be confused with a use that I introduce later, where 'unity' is used to describe a *type* of entities. The latter use I will distinguish from the former one by capitalizing the 'u'. Thus in the former sense, it can be said that an entity *has* unity, or that it does not. In the latter sense, we might say that an entity *is* a Unity, or that there are many Unities in the room. In particular, it must not be assumed that any entity which has unity is a Unity, or vice versa that any entity that is a Unity has unity.

Let us say that when comprised entities are closely held together, in this sense, there subsist between these entities relations of 'integrity', or, equivalently, that they are 'integrally' related to one another.⁸ Thus a comprising entity is unified iff the entities it comprises are integrally related to one another.

According to some authors, such as Simons and van Inwagen, unity is a constitutive feature of individuals.⁹ A concrete comprising entity is an individual, according to such a view, only if the comprised entities are integrally related to one another. We may describe this as the 'Unified Individuals View' (UIV):

UIV x is an individual only if the entities comprised by x are integrally related to one another.

According to other authors, such as Goodman and Lewis, this is rejected: a comprising entity might be an individual without being unified, i.e., without its comprised entities being integrally related to one another.¹⁰

Versions of the Unified Individuals View are distinguishable from one another in two principal respects. One concerns the question, which relations count as relations of integrity. The other concerns the question, whether there exist concrete entities which are *not* unified (and thus do not belong to the category of individuals).

Regarding the first question, authors may agree that unity, constituted by relations of integrity between comprised entities, is an essential feature of individuals, and yet disagree in their account of integrity. Relations of contact or attachment (i.e. *are in contact with one another*, or *are attached to one another*),¹¹ for example, would seem at first glance to be attractive candidates for being integrity relations. However, reflecting that this would imply, for instance, that 'John's family' would (implausibly) designate an individual if the family members were in contact with, or attached to, one another, theorists have often sought more

⁸ Simons uses 'integrity' in my sense of 'unity'. See Simons 1987, 326ff.

⁹ See e.g. Simons 1987, 290-293; van Inwagen 1987.

¹⁰ See Lewis 1991, 79-81; Goodman 1956.

¹¹ A more complete discussion would require a precise explanation of what is meant by saying of some entities that they are in contact with one another, or that they are attached to one another. See, for example, van Inwagen's (1987) account of 'contact'. Such elaboration, however, is unnecessary for our present purposes.

sophisticated proposals regarding integrity relations.¹² Thus, according to one among these more sophisticated types of account, integrity relations are relations of ontological dependence between the comprised entities. According to another, they are relations of functional dependence between the comprised entities.¹³

It should be noted that by deciding between UIV and its denial, and further by proposing (in the former case) an account of relations of integrity, an author is at the same time ruling on the extension of 'individual', offering a view regarding the question, which entities are really, rather than merely *prima facie*, individuals.

Regarding the second question, authors may agree that unity is an essential feature of individuals, and yet disagree as to whether it is an essential feature of concrete entities in general, or, in other words, whether concrete entities exist which are not unified, and thus not individuals. Burge uses the term 'aggregate' to designate such concrete non-individuals; Simons uses the term 'plurality', and further uses the terms 'class' and 'group' to designate types of plurality.¹⁴

Schematically, therefore, we may distinguish between the following three types of view regarding concrete comprising entities:

- (1) UIV rejected
- (2) UIV accepted; the existence of concrete non-individuals rejected
- (3) UIV accepted; the existence of concrete non-individuals accepted

Views of type (2) or (3) further differ from one another by the account they give of unity (or integrity).

We note that both according to (1) and according to (3) there exist concrete entities which are not unified, with the only difference between these views being that according to (1) such ununified entities are individuals, while according to (3) they are non-individuals. The question arises, what the difference between these positions consists in, terminology aside. According to Simons, although both

¹² See van Inwagen 1987, 26ff.

¹³ See Simons 1987, chs. 8 and 10; van Inwagen 1987 and 1990, Section 9.

The notion of an 'organic whole' is sometimes understood as that of a whole whose parts are ontologically dependent on one another, and sometimes as that of a whole whose parts are functionally dependent on one another. Ontological dependence and functional dependence are discussed in Chapter 5.

¹⁴ See Burge 1977; Simons 1987, 144-148; 331-333.

individuals and pluralities are concrete entities which comprise other entities, the character of the relation between comprised and comprising entities is different in the two types of cases. For example, when an individual comprises many entities, then any proper part of any of those entities is a part of the individual. By contrast, when a plurality comprises many entities, then any proper part of any of those entities is *not* a part of the plurality.¹⁵ Indeed, the difference between these relations is such that only in the former case are we justified, according to Simons, to describe each of the comprised entities as a 'part' of the comprising entity. (3) differs from (2), then, on Simons's account, in taking there to be two fundamental types of comprising relations which pertain to concrete entities, rather than only one such type.

For Simons, therefore, the predicates 'is an individual' and 'has parts' have the same extension. It suits the purposes of my argument better, however, not to assume at this point the equality of extensions of these predicates. I shall assume that the distinction between concrete individual and concrete non-individual corresponds simply to the distinction between unified and ununified entities, and I leave aside, for the time being, the question whether this distinction corresponds to the distinction between entities that have parts and entities that do not^t, respectively.

In considering concrete entities which have parts, I shall use the broader term 'entity', rather than 'individual', thus not ruling out in advance the possibility that concrete non-individuals should have parts. Note, however, that to allow that non-individuals might have parts is not to assume that comprising relations are always relations between wholes and parts. Thus my assumptions so far do not conflict, for example, with the claim that some concrete entities have members but not parts. The question whether some concrete entities comprise entities which are not their parts is not a central concern of the present work. It will, however, be further touched upon as this Part proceeds.

To summarize regarding my use of 'entity': unless otherwise made clear in the context, I shall use 'entity' neutrally as designating both concrete individuals and concrete non-individuals, but not abstract entities of any sort. I shall assume that some entities have parts, without presupposing either that only individuals have

¹⁵ See Simons 1987, 148-53 and 162-9.

parts, nor that comprised entities are necessarily parts of the entities which comprise them.

In speaking of concrete comprising entities, - whether controversial or uncontroversial concrete comprising entities - I shall be speaking therefore about concrete entities which are either unified or ununified. I shall assume, however, that if the entities *comprised* by the entity under consideration are themselves comprising entities, then they are uncontroversial comprising entities. A collection of pebbles is a controversial comprising entity. Each of the comprised pebbles is an uncontroversial comprising entity.

Furthermore, I shall confine my discussion to entities of which it makes sense to ask, how many of them there are (in a certain place, or answering a certain condition). It makes sense to ask how many pebbles there are on the beach. And it makes sense to ask how many *collections* of pebbles there are on the beach (although this second question may admit of several incompatible interpretations, which correspond to different assumptions that might be adopted concerning the way to count collections). It does not make sense to ask "how many water there is in the sea", or (unless one takes 'water' in the sense of 'a body of water') "how many waters there are in the sea". Rather, one may ask how *much* water there is in the sea.

This means that I shall be ignoring the ontological category of stuffs or masses. To the extent that I shall have occasion to mention such things as lumps of gold or heaps of sand, I shall be treating them under the tentative assumption that theories devised with organisms, pebbles, and collections of pebbles in mind are applicable to such lumps or heaps. This is not because it is my considered opinion that stuffs do not require separate treatment, but rather because it would unacceptably complicate and enlarge this work, if such separate treatment was attempted here. I cannot, at present, see that the validity of my main argument will be affected by this simplifying assumption. It seems to me that bringing considerations to bear regarding the characteristics of stuffs will at most have the effect of restricting the generality of my conclusions. I trust that they will be of interest even if taken to be of less general validity.

Section 1.2

Ways of Being One

1.2.1 *Ambiguities Regarding 'is one'*

The notions of a collection, a whole, a sum and a Unity¹⁶ are at the centre of the present work's concerns. They can all be described as notions of comprising entities. They will be introduced in turn, and the relations between them will be clarified as we proceed. One fundamental component of such a clarification needs to be treated already at this stage, however, before we turn to the specific discussion of any of these notions. This concerns the ambiguity which attaches to the claim that a comprising entity is 'one', or 'one entity'.

Authors, both ancient and modern, have felt the contrary pull of describing comprising entities (of some types, at least) as being at the same time 'one' and 'many' or 'not one'.¹⁷ Plato, for example, explicitly emphasizes, in one context, that a whole is one, and in another context that it is many.¹⁸ Perhaps less explicitly, what Aristotle says about heaps seems to suggest that he takes a heap both to be one and many.¹⁹ David Lewis's account of mereological fusions seems to suggest

¹⁶ Again, it must be emphasized that 'Unity' should not be confused with 'unity' - see footnote on this point in 1.1.3.3.

¹⁷ I shall be assuming that in all the relevant contexts 'one' and 'many' are treated as contraries, so that if x is one, in some specified sense, then in a corresponding sense x is not many. It is because of the status of 'one' and 'many' as (*prima facie*) contrary predicates, that describing an entity as being both one and many is puzzling and requires an account whereby different senses of 'one' (and correspondingly of 'many') are distinguished.

¹⁸ On the one hand it is argued in *Parmenides* 137c4-d3 that something's being a whole implies that it consists of parts and "is in that way many and not one" (and on the grounds of this implication it is argued that the one itself "cannot have any parts or be a whole"). And in *Theaetetus* 204a7-205a9 it is argued that "when a thing has parts, the whole is necessarily [sc. is necessarily identical to] all the parts", and so, presumably, since they are many, it is many.

On the other hand the whole is described in *Parmenides* 157d8-e1 as "some single form and one thing" and as "a complete one which has come to be out of all [sc. the parts]" (The translation of the last two excerpts is more literal than Cornford's "a single entity or 'one'" and "a complete 'one' composed of all", respectively). And a similar point is made in *Sophist* 245a1-3.

¹⁹ On the one hand, it is said in *Metaphysics* 1040b5-9 of "the parts of animals ... and earth and fire and air" that "none of them is one, but they are like a heap before it is fused by heat

that he takes a fusion to be one, in one respect, but many, in another.²⁰ Simons explicitly states that pluralities are in a sense one, in another many.²¹ In *The Principles of Mathematics* Bertrand Russell urged the notion of what he called 'the class as many', and by contrast with the types of positions just mentioned insisted with regard to such a class that "the many are only many, and not also one".²² However, more than one commentator has found Russell's position here unintelligible.²³ And it must be noted that Russell himself pointed out in the chapter 'The Contradiction' that in some respect the class as many is, after all, one.²⁴

This is not to say, of course, that these well respected authors are all guilty of inconsistency. Examining their positions, one finds, indeed, different grounds for claiming that an entity is one rather than many. I shall consider features which

and some one thing is made out of the bits" (see also *Metaphysics* 1045a8-10). Thus it would seem that in some respect a heap (before it is fused) is not one.

On the other hand it seems to be conceded in *Metaphysics* 1015b36-1016a3 that things which are continuous, such as the parts of the body, are legitimately called one. So it would seem that in some respect a heap - provided that it is continuous - is one.

It might be suggested that this apparent tension in what Aristotle says about heaps is to be resolved by saying that Aristotle uses the term 'heap' in some contexts in an ordinary sense (according to which the term may apply to continuous entities), while in other contexts he uses the term in a technical sense (Miller 1978, 108-9, for example, claims that such a distinction between senses must be drawn).

However that may be, note that in the course of presenting a modern version of Aristotle's account, Scaltsas affirms a tension as regards an aggregate's (concerning the use of either 'heap' or 'aggregate' in translating the Greek ' *Soros* ', see footnote 1 above) being both one and many, which *cannot* be resolved by reference to such a distinction between senses. For he says that "the aggregate is not a single entity; it is the many things that it is the aggregate of" (Scaltsas 1994b, 66). And yet he claims that "... any number of things can be unified into a group just by including them in a group The ground for such unity is convention, and the resulting oneness is entirely compatible with the plurality that the things in the group constitute" (Scaltsas 1994a, 109).

²⁰ He describes the relation of the many parts to their fusion as a 'many-one relation' (see Lewis 1991, 82). This seems to imply that the fusion is one. On the other hand, he says that "mereology is innocent in a different way [i.e. different from the way plural quantification is "innocent"]": we have many things, we do mention one thing that is the many taken together, but this one thing is nothing different from the many" (Lewis 1991, 87). I shall discuss Lewis's position in detail in 7.2.3 below.

²¹ "... a plurality is in one sense one, and in another sense many. It is one class (as distinct from other classes), but at the same time it is (i.e. is composed of, has as members) many individuals." (Simons 1987, 151-2).

²² See Russell 1903, 76.


²³ Black 1971, 620-1 takes Russell's account of the distinction between class as many and class as one to be an example "of what Berkeley called the 'darkness and confusion of mathematics'". See also Grossmann's (1983) discussion of classes.

²⁴ Russell 1903.

provide such grounds as respects in which an entity is said to be one. Thus one finds that different respects may be distinguished in which an entity might be said to be one rather than many. Before turning to the more important of these respects, let us set aside other ways in which things are said to be one, which are of less interest in the context of a comparative characterization of comprising entities.

1.2.2 *Being One (A): linguistic oneness*

If x is a horse, then clearly x is *one* horse.²⁵ It might then be said that x is *one*, where this is meant simply as a way of saying that x is one horse (a less awkward way of expressing x 's being one in this respect would be to say that it is one *in number*). In this sense, a house is one, a pebble is one, a collection (say, of pebbles) is one, a heap is one, etc. In general, if ' F ' is a term such that the sentence ' x is an F ' is a (correct) description of x , then it is the case that x is one F ; and so, in the sense we are considering, it is the case that x is one. Since expressions used apparently to refer to comprising entities, such as 'whole', 'heap', 'sum', 'class', 'collection', etc., are all expressions of this type, it follows that whenever one of these terms is correctly predicable of x , then x is one in this sense.

For x to be one in this sense seems to be simply a consequence of the linguistic fact  that a descriptive term in the singular is correctly predicable of x . To say that x is one in this sense is not so much to say something about the nature of x , as it is to say something about the way x is treated according to the conventions of the English language.²⁶ Therefore, I call the feature of being one in this sense 'linguistic oneness'.

²⁵ One plausible way to interpret the claim ' x is one horse', is to say that it is equivalent to the claim 'there is one and only one horse y to which a certain description D applies, and $x = y$ '. It can be formally expressed without using the term 'one' as follows:

x is one horse iff
 for some y , for some description D ,
 (y is a horse and D applies to y , and for all z , if z is a horse and D applies to z , then $z=y$,
 and $x = y$).

²⁶ Frege's discussion in *The Foundations of Arithmetic* suggests that something's having the characteristic of being one in number (compatibly with having the characteristic of being,

Correlatively with the notion of linguistic oneness, a notion of *linguistic manyness* may be distinguished. If x and y are horses, then clearly x and y are *many* horses. We might say that x and y are many, meaning by this that for some term ' F ' the sentence ' x and y are F s' is a correct description of x and y . We do not assume that being linguistically one is incompatible with being linguistically many. For example, if it is supposed that z , a sum of sand grains, is identical to two distinct sums of sand grains x and y (taken together),²⁷ then since z is linguistically one, and x and y are linguistically many, and z is identical to x and y , it follows that z is both linguistically one and linguistically many.

Note that for x to be linguistically one does not imply that a *syntactically singular* term is predicable of it. Simons notes a threefold distinction between (1) syntactically singular, (2) syntactically plural, and (3) syntactically indeterminate terms.²⁸ This distinction has to do with the form of *verbs* combining with such terms to form grammatical sentences. The criterion is whether verbs combining with the term in question are singular, plural, or either singular or plural. Thus 'Socrates' is syntactically singular because we say 'Socrates *is* singing'; 'Lennon and McCartney' is syntactically plural because we say 'Lennon and McCartney *are* philosophizing'; and 'the orchestra' is syntactically indeterminate, because we may say either 'the orchestra *is* playing' or 'the orchestra *are* playing'. If x is an orchestra, then it need not be the case that a syntactically singular term is predicable of it (we have just noted that 'orchestra', for example, is not syntactically singular, but rather syntactically indeterminate). However, if x is an orchestra, then x is one orchestra, and so it follows that x is linguistically one.

say, five in number) is to be explained with reference to the fact that a certain term in the singular (e.g. 'copse') is correctly applicable to it:

While looking at one and the same external phenomenon, I can say with equal truth both "It is a copse" and "It is five trees", or both "Here are four companies" and "Here are 500 men". Now what changes here from one judgement to the other is neither any individual object, nor the whole, the agglomeration of them, but rather my terminology. But that is itself only a sign that one concept has been substituted for another. (Frege 1884, 59)

In this context, at least, it seems that Frege understands the notion of being one in number as what I have called 'linguistic oneness'.

²⁷ Regarding such alleged cases of one-many identity, see comments in 1.2.6 below.

²⁸ See Simons 1987, 142-3. The examples I mention in this paragraph are Simons's.

1.2.3 *Being One (B): atomicity*

The claim that an entity is atomic is usually understood as the claim that it has no proper parts.²⁹ Since our discussion has not yet been confined to wholes and parts, and we are speaking of comprising entities more generally, it is useful to speak of a correspondingly more general notion of being atomic. Therefore, in the present context I take it that an entity is atomic if and only if it either comprises no entities at all, or it comprises only itself. The null class, for example, is atomic in this sense. And if a concrete entity has no proper parts then (assuming it comprises no entities which are not among its parts) it too is atomic in this sense. We see then that the more general notion of being atomic entails the narrower and more familiar one mentioned at the beginning of this paragraph.

Sometimes an entity will be said to be one on the grounds that it is atomic. Indeed, it seems natural to treat entities that are one in this respect as best deserving to be described as one, and thus as paradigms of being one. It seems that for this reason Plato claims that what he calls 'the one itself' cannot have parts, and that by having parts a thing is rendered (in some respect) many rather than one.³⁰

Regarding the types of entities with which the present work is concerned, however, all the competing views which I consider agree in treating such entities as being *not* atomic, in the general sense I have noted.³¹ The notion of atomicity³² will therefore be of marginal importance in our discussion.

²⁹ See, for example, Simons 1987, 16. I prefer the variant 'is atomic' to Simons's 'is an atom' here.

³⁰ See *Parmenides* 137c-d. That a thing which has parts can be one only in a more qualified way (than a thing which does not have parts is one) is also claimed in *Sophist* 245a-b.

³¹ Thus a man, for instance, works out to be *not* atomic, whether conceived of as (a) a distributive class of cells or (b) a collective class of cells or (c) a collection of cells or (d) a whole (or organic whole, or Gestalt) of which the cells are parts or (e) a classical sum of cells or (f) a neoclassical sum of cells or (g) a Unity of which the cells are either elements or pre-elements. These notions will all be explained in turn, and related to one another, as we proceed.

³² The term 'atomicity' is sometimes used to describe the feature of an entity whereby it comprises parts which are atomic. It should be emphasized that I use the term differently, to designate the feature of being atomic itself.

1.2.4 Being One (C): relational oneness

Sometimes 'one' is used not in the context of sentences of the form ' x is one' but rather in the context of sentences of forms such as ' x and y are one' (or more generally ' x_1, x_2, \dots, x_n are one'). The notion of being one is understood in the latter type of contexts as that of a *relation* between entities.³³ In most cases of this sort, to say that x and y (or, more generally, x_1, x_2, \dots, x_n) are one is to say either of the following things: (1) x and y are the same (either numerically, or in some particular respect), or (2) x and y compose (or, more generally, are comprised by) one entity, or (3) x and y are integrally related to one another. The notion of being one, as associated with either (1), (2) or (3), can be generically described as the notion of 'relational oneness'.³⁴

I note this notion of relational oneness here in order for it not to be confused with notions associated with use of the *monadic* predicate 'is one'. Any claim whereby a comprising entity is said to be one is a claim which utilizes the latter monadic predicate. And it is with accounting for different ways in which use of 'is one' is understood that we are concerned in the present Section.

This is not to say, however, that some of the notions of being one associated with use of the monadic 'is one' may not be importantly bound up with the notion of relational oneness. This will be particularly evident with regard to the notion of unity, presently to be discussed.

³³ The relation might either be dyadic, polyadic, or multigrade.

³⁴ Many of ^{the} ways of using 'one' which Aristotle surveys in *Metaphysics* V, 1015b16-1017a6, are associated with expressing relational oneness. Thus to say that Corsicus and musical are one (see 1015b17-34), that oil and wine are one (see 1016a18-23), that horse, man and dog are one (see 1016a24-32), that "those things, the thought of whose essence is indivisible and cannot separate them either in time or in place or in formula, are most of all one" (1016b1-3; see 1016a33-1016b11), that "things that are one in number are one in species, while things that are one in species are not all one in number" (1016b31-1017a3) - all these are cases involving a relational use of 'one'.

Non-relational uses of 'one' (i.e. as a *monadic* predicate) are discussed by Aristotle in three passages: 1015b35-1016a17, 1016b12-17, and 1016b18--30. The first and second passages discuss cases in which an entity is said to be one on the grounds that its parts are related to one another in a particular way. Relations which provide such grounds are what I have called 'integrity relations'. The notion of being one associated with an entity's having parts which are integrally related to one another is what I have called 'unity', to be discussed directly below. As for the third passage, what Aristotle says here seems to be of particular relevance to the notion of atomicity, which has been briefly explained above.

1.2.5 *Being One (D): unity*

Perhaps the most common way of understanding the claim that an entity is one, is to take it to mean that the entity is unified, i.e. possesses the feature of unity, which has already been discussed above (see 1.1.3.3) as what is considered by some authors to be an essential feature of individuals. It was explained that for a comprising entity to possess unity is for it to comprise entities which are integrally related to one another.³⁵

The emphasis on this respect of being one seems to owe much to Aristotle's extensive observations regarding the close association between the claim that an entity is one and the fact that the parts of the entity bear certain relations to one another.³⁶

An understanding of the notion of being one in terms of the notion of unity is suggested, for example, by van Inwagen's discussion of integrity relations in 'When are Objects Part'. He is inquiring which changes in the relations between many entities could correspond to a transition from a state of affairs in which the many exist, but no entity which comprises them exists, to a state of affairs in which the many exist and in addition an entity (i.e. *one* entity) which comprises them exists. He asks, for example, whether when "you and I shake hands ... Does a new thing at that moment come into existence" (and answers in the negative, regarding this and other instances of the relation of contact).³⁷

This sort of understanding of the notion of being one is similarly suggested by Simons's discussion of relations of integrity, when he says that the claim that an individual exists is justified on the basis of facts about relations of integrity which hold between entities comprised by the individual.³⁸ If the comprised entities are not integrally related, then we are only justified in claiming that a comprising entity

³⁵ It is clear that the notion of unity is closely connected with the notion of relational oneness which was discussed above. For a comprising entity is one, in the sense associated with unity, if and only if the entities it comprises are one, in the sense associated with relational oneness.

³⁶ See footnote 34.

³⁷ See van Inwagen 1987, 28.

³⁸ See Simons 1987, 331.

of the type he calls '*plurality*' exists, rather than an individual. And as noted above, a plurality according to Simons is not one in the way an individual is.³⁹

It should be noted that talk about oneness that comes in *degrees* is most reasonably interpreted as talk about unity. It is natural to compare ways in which entities are related by saying that some are *more closely related* to one another than others. Relations of integrity might in this way be loosely taken as being of different degrees, so that those of a higher degree lend a higher degree of unity to the associated comprising entity. I shall not attempt to assess the appropriateness of such talk of degrees of unity.

1.2.6 *Being One (E): monadicity*

In *Theaetetus* 203c4ff., Plato contrasts the notion of a comprising entity which is identical to the entities it comprises (collectively),⁴⁰ with that of a comprising entity which is not identical to the entities it comprises (again, collectively). Describing the second type of comprising entity, he emphasises a sense in which it is one by saying that it is "some single form produced out of [the elements which it comprises], having its own single nature - something different from the letters."⁴¹

This suggests a feature on the grounds of which an entity might be described as one: An entity y is one, in this sense, if and only if for all x_1, x_2, \dots, x_n (all distinct from one another, and such that n is greater than 1), y is not identical to x_1, x_2, \dots, x_n

³⁹ See footnote 21 above.

⁴⁰ I use the expression 'distributive identity' and 'collective identity' (and cognates) to make a contrast which can be roughly stated as follows:

y is identical to x_1, x_2, \dots, x_n *distributively* iff y is identical to *each* of those entities; y is identical to x_1, x_2, \dots, x_n *collectively* iff y is identical to those entities "taken together". I shall discuss this contrast in greater detail in Section 7.2 below.

When Plato is examining the thesis that 'SO' is identical to 'S' and 'O', he clearly does not intend the radically implausible thesis that 'SO' is identical to 'S' and 'SO' is identical to 'O'. The thesis he is examining must, it seems, be that 'SO' is identical to 'S' and 'O' where these are somehow taken together, i.e., collectively.

⁴¹ *Theaetetus* 203e3-5. The word 'single' appearing here in the Burnyeat (Levet) 1990 translation renders first '*hen*', then '*mia*', the normal masculine and feminine forms for 'one' in Greek.

(collectively). In particular, if y comprises the x s, and y is one in this sense, then it is not identical to the x s (collectively).

I shall describe an entity which possesses this feature as 'monadic'.⁴² Plato's contrast between two conceptions of a comprising entity in the *Theaetetus* is a contrast between a conception of a non-monadic comprising entity, and a conception of a monadic comprising entity. Anyone who takes it, as do Doepke's One-Many theorists, and Simons does,⁴³ for example, that one-many identity claims are improper and thus unacceptable in principle, will take it that *all* comprising entities are monadic. I shall be arguing at a later stage, however, not only that such claims are acceptable in principle, but also that sums as conceived of in classical mereology are indeed non-monadic. I shall further argue that it is questionable whether theories which diverge from classical mereology, and yet retain fundamental features of the notion of a sum in their conception of wholes, have been successful in elaborating a coherent model for monadic concrete comprising entities. The notion of a Unity, which I introduce in Part III of the present work, seems to offer a more promising model for monadic comprising entities.

It should be noted that there would seem to be no definitional connections between the notions of monadicity and unity which would allow the reduction of one of the notions to the other. The questions, whether an entity is monadic, and whether it is unified, seem to be independent questions. The impression that they are independent is encouraged by the fact that when Lewis considers the question whether a sum is identical to its parts (that is, whether a sum is non-monadic),⁴⁴ the question of relations between the parts does not enter into his considerations. And the impression is similarly encouraged by the fact that Plato's discussion of the contrast between non-monadic and monadic comprising entities in *Theaetetus* makes no mention of relations between the relevant comprised entities.

⁴² I use the term without intending to connote Leibniz's notion of a 'monad'. Note also that use of 'monadic' to describe comprising entities is intended to have nothing to do with use of the term to describe a type of relation (a type which contrasts with that of polyadic relations).

⁴³ See Doepke 1984, 46-7; Simons 1987, 212 ;

⁴⁴ See Lewis 1991, 81-87. Note that Lewis uses 'fusion' for the classical mereological notion which I shall be calling 'sum'. The notion of a sum is discussed in detail in Part II.

Note, however, that an entity's being atomic implies that it is monadic. On the other hand, an entity's being monadic does not entail its being atomic.

1.2.7 *Being One (F): singleness*

There is a fundamental respect in which an entity might be said to be one, which is not captured by the accounts provided for any of the features so far mentioned.

Suppose we wish to consider, for example, the question whether a certain sum of pebbles is identical to the pebbles of which it is the sum.⁴⁵ We may imagine that the pebbles are widely dispersed on some beach. Let us assume that the number of pebbles is greater than 1. Thus if the sum of pebbles is identical to the pebbles, it is non-monadic. So I shall take an equivalent formulation of the question to be, whether the sum is non-monadic.⁴⁶

Consider ways in which the sum of pebbles is said to be one, apart from the respect under investigation, i.e. that of monadicity. Let us assume that the sum is not one, in the sense that it is not unified - the relations between the pebbles are not relations of integrity. Furthermore, since the sum comprises at least two distinct pebbles, it is also not one, in the sense that it is not atomic. Clearly, however, the sum is one in the sense that it is linguistically one. For it is obviously correct to say that the sum is one sum.

Now it would seem, intuitively, that the meanings of the terms 'the sum of pebbles' and 'the pebbles' are distinct. So that if it turned out that the sum of pebbles is non-monadic, and so identical to the pebbles, we should explain the relation between the two terms (in respect of their semantic properties) as analogous to the relation between 'the morning star' and 'the evening star'.

⁴⁵ a. I introduce the issue of singleness in terms of the example of a comprising entity designated by 'sum'. Precisely the same problems, however, arise in connection with other types of comprising entity - collections, wholes, etc.

b. Unless otherwise stated, statements of identity between one entity and many entities are to be understood in the *collective* sense

⁴⁶ The questions are equivalent assuming that if the sum is not identical to the pebbles then *a fortiori* it is not identical to any other entities which are greater than 1 in number.

Otherwise the answer to the question whether the sum of the pebbles is identical to the pebbles would be obviously, and uninformatively, affirmative. But it isn't. To assume that it is *obviously* affirmative would be to fail to make sense, for example, of David Lewis's searching discussion of the question in *Parts of Classes*.⁴⁷

Moreover, it seems that one aspect of this difference of meaning between 'the sum of pebbles' and 'the pebbles' is the following: it is part of the meaning of 'the sum of pebbles' that what it is used to refer to is *one*, in a way that it is *not* part of the meaning of 'the pebbles' that what it is used to refer to is one.⁴⁸ However, the notion of being one required here could not be that of atomicity (for the sum comprises more than one pebble). Nor could it be that of relational oneness (for the sum of pebbles is said to be one, not the pebbles). Similarly, it could not be the notion of unity (for we have assumed that the sum of pebbles, in the particular case we are considering, is not unified). Nor, again, could it be the notion of monadicity (for it is precisely the question whether the sum of pebbles is monadic which is under investigation; to assume that it is in setting out the problem would be to beg the question).

Of the features so far explicated, in association with claiming that an entity is one, only linguistic oneness remains⁴⁹ as a candidate for helping to account for the difference in meaning between 'the sum of pebbles' and 'the pebbles'. It seems, however, that this notion too is not able to bear the weight of the task. Consider, for comparison, the contrast between 'the orchestra' and 'the players'. The expression 'the orchestra' is syntactically indeterminate, so that in some contexts it combines with verbs in the *plural* to form grammatical sentences (while combining in other contexts with verbs in the *singular* to form grammatical sentences). It seems that in such contexts the meanings of the two expressions do *not* differ as regards being one. In a context in which we say that the orchestra *are* playing, the meaning of 'the orchestra' does not seem to connote a oneness which 'the pebbles' does not. So the difference we are looking for cannot be explained by reference to linguistic oneness.

⁴⁷ See Lewis 1991, 81-7.

⁴⁸ Lewis makes such an assumption when he urges (as a reason for finally denying that a sum is identical, strictly speaking, to the parts which compose it) that "what is true of the many is not exactly what's true of the one. After all they are many while it is one" (Lewis 1991, 87).

Something can be linguistically one without displaying the oneness we are inquiring about.

It appears then that the fact that the sum of pebbles is linguistically one cannot explain the difference in meaning between 'the sum of pebbles' and 'the pebbles', as regards being one. Therefore, we are led to conclude that a consideration of the question whether the sum of pebbles is monadic presupposes that the sum of pebbles is one, in a sense which is distinct from that associated with each of the notions discussed so far.

I shall describe an entity that is one in this other sense as being 'single'. The question that faces us is how to account for this notion of a single entity, which I have so far characterised only negatively (i.e., by saying that being single is not the same as being linguistically one, or atomic, etc.). The account I wish to propose is a rough and tentative one, and yet I hope it suffices to show that the notion of a single entity is not a hopelessly obscure one. In any case, I do not pretend to have shown conclusively that we are forced to acknowledge some such notion. I must emphasize, however, that the larger argument of the present work will not depend on the assumption that such a notion must be acknowledged, nor, in particular, that the account I offer for this notion is an adequate one.⁴⁹

I propose, then, to account for the notion of a single entity as follows. Tokens of definitely referring expressions, such as 'the horse', or 'those houses', can be characterised as bearing a *semantic relation* to what they are used to refer to. I shall assume that such semantic relations always belong to one of two types: that of one-one relations, or that of one-many relations. That is, with respect to the positions they occupy in the structure of such a semantic relation, the referring expression is always one, and what is being referred to is either one or many.

⁴⁹ It should be remembered, however, that if the notion singleness is rejected, then when we compare, as regards being one, what is designated by different expressions for comprising entities - for example 'collection of cells' and 'organism' - we can only contrast them with regard to one of the features so far accounted for. So that if (as might be plausibly claimed), for example, both are linguistically one, both are non-atomic, both are equally unified (since they comprise the same cells, and relations of integrity between the cells determine the unity of any entity which comprises them), and both are monadic (as I have noted above, on some views any comprising entity must be deemed monadic), then there would seem to be no sense in which the organism is one, according to which the collection of cells is not one.

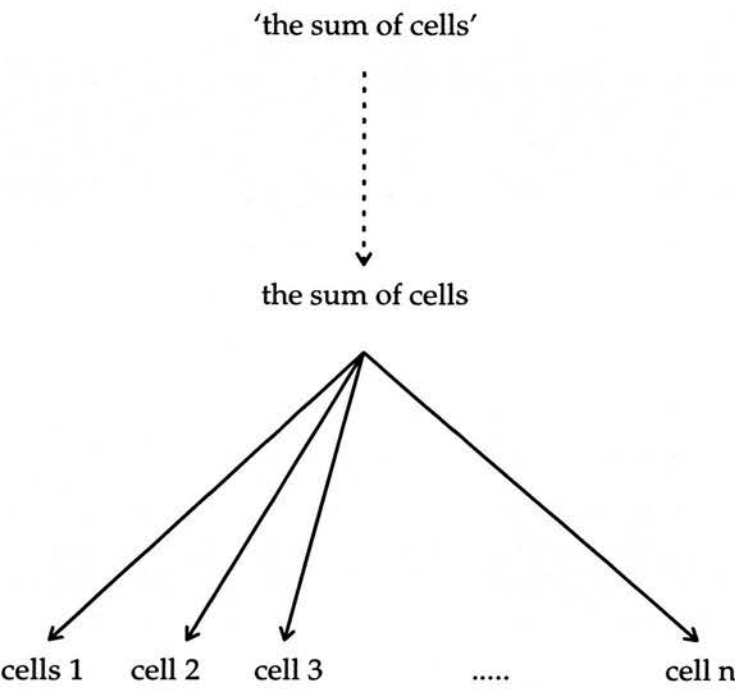
I am assuming, in particular, that neither the referring expression, nor what is being referred to, can be *indeterminate* in this respect: neither of them can be either indeterminately one or indeterminately many. This assumption seems to be very natural, and is principally supported by the obscurity which would be involved in the alternative of allowing an indeterminateness in this respect.

Expressions (by 'expression', in the context of the present Subsection, I shall mean '*token* expression', unless otherwise emphasized) associated with a one-one semantic relation I shall describe as 'one-one expressions', whereas those associated with a one-many relation I shall describe as 'one-many expressions'. Paradigmatic examples of one-one expressions may be found among expressions involving terms designating atomic entities of certain types, in the singular, such as 'the electron'. As regards one-many expressions, corresponding paradigmatic examples would be expressions involving terms designating atomic entities in the *plural*, such as 'the electrons'.

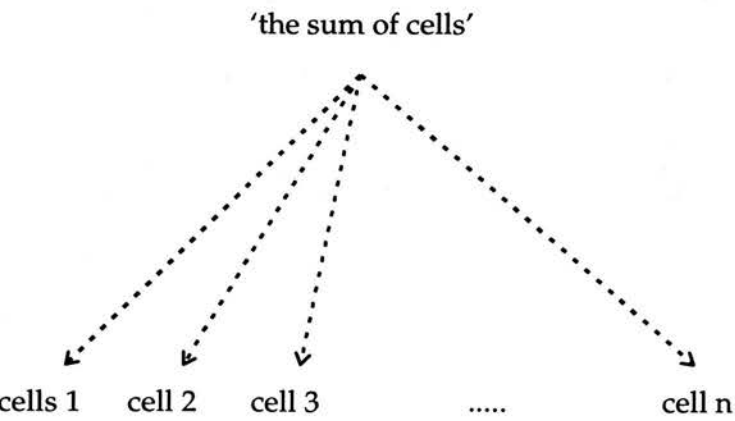
The notion of a single entity can now be explained on the basis of these assumptions regarding the semantics of definitely referring expressions. I propose that an entity is single if and only if definite reference to it can be made by means of a one-one expression. Contrastingly, an entity will be described as 'multiple' if and only if definite reference to it can be made by means of a one-many expression.

Thus suppose definite reference to a certain sum of pebbles can be made using the expression 'the sum of pebbles'. If this expression is one-one, then the sum of pebbles is a single entity. If the expression is one-many, then the sum of pebbles is a multiple entity. In the former case, the relation between the expression 'the sum of pebbles' and the many pebbles is *mediated* by the comprising entity, the sum of pebbles. The expression is *semantically related* to the sum of pebbles, and the sum of pebbles *comprises* the pebbles. In the case of 'the pebbles', there is no such mediation; the expression is semantically related directly to the pebbles. The contrast between the cases can be illustrated as follows (semantic relations are represented by dotted arrows; comprising relations by continuous arrows):

Case 1: 'the sum of pebbles' is one-one
(implying that the sum of pebbles is a single entity):



Case 2: 'the sum of pebbles' is one-many
(implying that the sum of pebbles is a multiple entity):



Note that it has not been assumed that for an entity to be a single entity is incompatible with its being a multiple entity. In Case 1, it follows from our assumptions that the sum of pebbles is a single entity. It does not follow, however, that the sum of pebbles is not a multiple entity. For it to be a multiple entity, all that is required is that *some* one-many expression can be used to refer to it. It is not required that 'the sum of pebbles' is that expression.

If being single is not incompatible with being multiple, then we may consistently assume that the sum of pebbles is a single entity and the pebbles are a multiple entity, and yet the sum of pebbles is identical to the pebbles. Thus the expressions 'the sum of pebbles' and 'the pebbles' may have a different meaning (as regards being one), and yet they may be used to refer to the same entity. In this way the possibility of an informative, non-obvious answer to the question whether the sum of pebbles is identical to the pebbles is accounted for.

Having explained the notion of singleness in terms of the distinction between expressions that are one-one and expressions that are one-many, the question now arises whether a criterion is available by means of which we distinguish between expressions that are one-one from expressions that are one-many. Consideration of our example case of 'the sum of pebbles' and 'the pebbles' suggests a simple and attractive proposal.

We noted that what is referred to by means of the expression 'the sum of pebbles' is thought of as being one (in a way that, ultimately, we accounted for in terms of the notion of singleness). What is referred to by means of the expression 'the pebbles', by contrast, is correspondingly thought of as being many. It seems clear, on reflection, that this difference in the way we think about the referents of these expressions corresponds to a difference in the way we predicate things of them. Thinking of the referent of 'the sum of pebbles' as being one, in the relevant way, corresponds to saying that the sum *is* on the beach; thinking of referent of 'the pebbles' as being many corresponds to saying that the pebbles *are* on the beach.

The proposal, then, amounts to this: a token expression is one-one if and only if it is used (in the context of a grammatical sentence) in combination with a certain kind of verb (or, more generally, verb phrase) in the singular. It is one-many, on the other hand, if and only if it is used (in the context of a grammatical sentence) in combination with a certain kind of verb (or verb phrase) in the plural.

The unspecific qualification 'certain kind of' is meant to exclude a true statement such as 'the sum is a sum' from being taken to indicate that 'the sum' is here one-one, and so as evidence that the sum is a single entity. The qualification can be partly fleshed out by saying that any case in which the use of the expression in combination with a verb in the singular is guaranteed merely by the *linguistic oneness* of the entity referred to should not count as evidence for the referring expressions being one-one. One might admit that the orchestra is an orchestra, even in a context in which one insists on speaking of the orchestra using verbs such as 'play' in the plural. In such a context, it seems to me that no evidence is provided for the orchestra's being a single entity.

If the relevant distinction between kinds of verbs (or verb phrases) cannot ultimately be justified, then the qualification 'certain kind of' will have to be dropped, and complying with the criterion for linguistic oneness will guarantee complying with the criterion for singleness.

In what follows, I shall assume the correctness of this criterion without further argument. A critical examination of the suggestion would take us far afield into questions about the relations between syntactical and semantical structure of a language - an investigation which lies well outside the scope of the present work. The fact that this assumption is not strongly supported will be of little consequence, since, as noted before, my account of singleness does not play a crucial role in my larger argument. Its role is chiefly that of offering an additional perspective which might help grasp the significance of the claims I make later. Ultimately, if the account of singleness I offer is rejected, then the additional perspective is lost, but the soundness of the general argument is not affected.

This concludes my account of different respects in which a comprising entity might be said to be one. Armed with these distinctions regarding ways of being one, we are in a much better position to assess claims which contrast different types of comprising entity by saying that an entity of one type is "more one" than an entity of the other type. For we may examine each of the entities, feature by feature, and see in which respect each of the entities is one. If they do not differ either as regards linguistic oneness, or atomicity, or unity, or monadicity, or singleness, then

we shall conclude that initial impressions notwithstanding, they do not differ with respect to being one.

I arrive at precisely such a conclusion, for example, in comparing collections with sums. It might be assumed, at a first glance, that a sum, as conceived of in classical mereology,⁵⁰ is “more one” than a corresponding collection (i.e. a collection whose members are identical to entities of which the sum is a sum). However, according to a plausible account of the notion of a collection, which I outline later in this chapter, a collection is one in any respect in which a corresponding sum is one.

I shall be comparing three general conceptions of wholes, in the course of Parts II and III of this work. The availability of clear accounts as regards respects in which an entity can be said to be one helps us to clarify the relations between these conceptions. It turns out that monadicity, rather than unity, singleness, atomicity or linguistic oneness, is a crucial feature in connection with such a clarification. I shall argue that sums, as conceived of in classical mereology, are non-monadic entities. Many of the advantages of what I call the ‘neoclassical’ conception of wholes, over the classical one, have to do with precisely those features of neoclassical sums which render them monadic. However, these are at the same time features which are highly problematic. I shall cast doubt on the view that neoclassical mereology offers us a satisfactory conception of monadic wholes. The theory of Unities, which I introduce in Part III, offers an alternative conception of a monadic comprising entity, which is free from many of the difficulties afflicting the neoclassical conception.

⁵⁰ Classical mereology is introduced in chapter 4 below.

Section 1.3

Plural Quantification

A study of comprising entities has at the centre of its concerns a relation between many comprised entities and one entity which comprises them. While I have assumed the possibility, in principle, that the comprised entities will be identical to the comprising entity, one cannot sensibly investigate the relation between comprised and comprising entities without carefully distinguishing between talk about the former and talk about the latter. In some contexts one may alternate between the use of phrases like 'the pebbles are nicely arranged' and 'the pebble collection is nicely arranged' without engendering misunderstandings. In a context in which what one is concerned with is precisely the relation between the pebbles and the pebble collection, it is obvious that one cannot do so.

Moreover, a study of comprising entities requires attention not only to things said about *each* one of the comprised entities, but also to things said about *all* of them, claims which cannot be directly rephrased as claims about each of the comprised entities. The claim that those four people weigh a thousand pounds is an example of such a claim, if it is understood in the sense according to which the weight of the people *taken together* is a thousand pounds.

Such claims are most naturally expressed by means of sentences which contain *plural terms*, i.e. referring terms which combine (only) with verbs in the plural, such as 'Sarah and Suzy' or 'the man's bodily organs'. Singular terms, by contrast, are terms which combine (only) with verbs in the singular. Standard predicate calculus makes it possible to express formalized sentences which contain singular terms but not plural terms. To facilitate the expression of formalized sentences which contain plural terms, the standard calculus may be supplemented with the means for plural quantification, that is, with plural variables and quantifiers.⁵¹

⁵¹ The exposition that follows is much indebted to van Inwagen's discussion in *Material Beings*, 1990, Section 2.

Thus in addition to

singular variables: x, y, z, u, v, w

We have

plural variables: the xs , the ys , the zs , etc.

The expressions 'the xs ', 'the ys ', etc. are not to be treated as composite expressions, any more than the expressions for singular variables are. For our purposes it is best to think of them as plural pronouns, used in the same way as 'they' or 'them'. It will be convenient to allow the plural variables to be written in an alternative way in some contexts: 'those xs ' instead of 'the xs ', etc. Note that 'the xs ' is not taken to be constructed using ' x '. The fact that one of the *marks* used in writing the former is the mark used in writing the latter is to be treated as a typographical coincidence.

Using a singular variable, we might say ' x is in the house'. Similarly, using a plural variable, we might say 'the xs are in the house'. More generally, 'the xs are F ' (where ' $are F$ ' is a predicate). The expression 'is one of' is used in its natural sense, and so I will take it to be clear how sentences of the form ' x is one of the ys ' are to be understood. In terms of the relation *is one of*, the following useful types of sentence can be defined:

each of the ys is $F =_{\text{def.}}$

for all z , if z is one of the ys , then z is F

the xs are among the $ys =_{\text{def.}}$

for all z , if z is one of the xs then z is one of the ys

the xs are properly among the $ys =_{\text{def.}}$

the xs are among the ys , and for some z , z is one of the ys but not one of the xs

the xs are (severally) identical to the $ys =_{\text{def.}}$

for all z , z is one of the xs iff z is one of the ys

An important ambiguity must be noted in connection with sentences of the form 'the x s are F '. According to one reading, which I call the 'distributive' reading, 'the x s are F ' is equivalent to 'each of the x s is F ' (see the first of the definitions above for explanation of 'each of ..'). According to the other reading, which I call the 'collective' reading, 'the x s are F ' is not equivalent to 'each of the x s are F '.

For example, if the sentence 'the four people weigh a thousand pounds' is read distributively, it is taken to mean that each of the four people weighs a thousand pounds. If, on the other hand, it is read collectively, it is taken to mean that the (arithmetical) sum of the four people's weights is equal to a thousand pounds. Note that claims such as 'the x s make up y ' and 'the x s compose y ' are generally read in a *collective* sense.

Turning to quantifiers, we have

singular existential quantifiers : 'for some x ', 'for some y ', etc.

and

singular universal quantifiers: 'for all x ', 'for all y ', etc.

In addition, we have

plural existential quantifiers: 'for some x s', 'for some y s', etc.

and

plural universal quantifiers: 'for any x s', 'for any y s, etc.

Examples:

The sentence

- (1) someone weighs a thousand pounds

is formalized as

- (1') for some x , x is a person and x weighs a thousand pounds

The sentence

- (2) some people weigh a thousand pounds

is ambiguous, however.

It might be formalized as

(2') for some xs , each of those xs is a person and each of those xs weighs a thousand pounds

or as

(2'') for some xs , each of those xs is a person and those xs weigh a thousand pounds (i.e., taken together)

The sentence

(3) all entities have sums

is again ambiguous.

It is either to be read as

(3') for all x , for some y , y is a sum of x

or as

(3'') for any xs , for some y , y is a sum of the xs

Finally, we should note the different types of predicates which occur in contexts of plural reference. Singular reference makes use of singular n -adic predicates, that is, n -adic predicates which contain n free singular variables. Such predicates express n -ary relations. Thus 'is larger than' is a dyadic predicate, containing two free singular variables, which expresses a corresponding relation whereby one thing is larger than another thing. Plural reference makes use of plural predicates in addition to singular predicates. These plural predicates are known as *variably polyadic predicates*, containing free plural variables. The relations they express are known as *multigrade relations*. These names are meant to indicate that such relations, in many cases, hold between variable numbers of entities. Thus the variably polyadic predicate 'are carrying', which features in the sentence 'some people are carrying a piano', expresses a relation between a variable number of entities (it applies, for instance, to cases where two people are carrying a piano, to cases where three people are carrying a piano, etc.).

Chapter 2

Types of Comprising Entities

2.1 Section

Collective Classes and Distributive Classes

An important step in the development of a theory of what I have called 'controversial' comprising entities (entities designated by terms such as 'class', 'manifold', 'collection', 'aggregate', etc.) is the clear distinction between talk about the many comprised entities, on the one hand, and talk about the entity which comprises them on the other hand. It seems natural to consider talk about a comprising entity simply as a peculiar type of talk about the many comprised entities, a talk in which the many are "spoken of as one". Indeed, it may turn out that the comprising entity is identical to the many comprised entities. However, the question whether this is so can only be considered with clarity if our thought of, and talk about, the former is clearly distinguished from our thought of, and talk about, the latter.

In the 19th Century more attention was gradually being paid to comprising entities, due to the emerging importance of the role they played in connection with the logical analysis of language and with work on the foundations of mathematics. This led to a recognition (for example, in the works of Cantor, Schröder and Frege) of the need for a distinction between talk about a comprising entity and talk about the many entities it comprises.

Once it is clear that talk about a comprising entity cannot be simply assumed to be a peculiar kind of talk about the many comprised entities, it is also clear that that existence of the comprising entity cannot without justification be supposed to be merely an aspect of the existence of the many comprised entities. It is then realized that, granted the existence of the many comprised entities, an *additional*

assumption is required to guarantee the existence of the comprising entity. The general assumption playing this role in Cantorian set theory, for example, is the unlimited Comprehension Principle.⁵² It is noteworthy that the question of the existence of such entities was not felt to be problematic (at least where the many comprised entities were not infinite in number).

In addition to laying down assumptions regarding conditions under which a comprising entity exists, the development of a theory of comprising entities involves the introduction of assumptions regarding the relations between comprised entities and entities which comprise them.

A particularly noteworthy development in this respect was the distinction, due principally to Frege, between two different types of comprising entity, corresponding to two different types of relations between comprised and comprising entities. In an article published in 1895,⁵³ Frege criticises Schröder for failing to distinguish between two notions of a class (German term: 'Menge') in his influential *Vorlesungen über die Algebra der Logik*. The first is the notion of a class as what Frege calls a "collective whole". In specifying such a notion of classes, we "do not bring them into connection with concepts."⁵⁴ To see what is meant by 'bringing them into connection with concepts' it is perhaps best to contrast this notion of a class with the other one discussed by Frege.

The second notion is that of a class as the *extension* of a concept. The extension of the concept *man* is an entity which is internally articulated, in accordance with the concept, in a way that the collective whole is not. A collective whole is an unarticulated occupant of a specified spatial domain. The difference between these two notions is highlighted by the fact that there are two ways in which an entity can belong to a class-as-extension: it can belong to it as a member, or as a subclass.⁵⁵ Whereas there is only one way in which an entity can belong to a class-as-collective-whole: it can only belong to it as a part.

⁵² See Machover 1996, 12.

⁵³ Frege 1895.

⁵⁴ Frege 1895, 87.

⁵⁵ As Frege notes (Frege 1895, 92), Husserl similarly observes that there are two ways in which an entity can belong to a class-as-extension, in his 1891 review of the same work of Schröder's work.

The relation *is a subclass of* is transitive, as is the relation *is a part of*. The relation *is a member of*, by contrast, is not transitive. Furthermore, both *is a subclass of* and *is a part of* may be assumed to be reflexive,⁵⁶ but not so the relation *is a member of*. In as far as a theory of classes is meant to serve as a tool in the formalization of logic, it is classes-as-extensions that are required, for it is only such classes that allow us to avoid conflating, for example, the logical form of 'Socrates is a human-being' with that of 'the Greeks are human beings'.

Frege's distinction can be thought of as marking a point of bifurcation, where modern theorizing on classes became separate from modern theorizing on wholes and parts. His classes-as-collective-wholes are predecessors of contemporary mereological sums, and his classes-as-extensions are predecessors of contemporary classes and sets.

Classes-as-extensions were soon to meet grave difficulties centring on the well-known Russell's Paradox, which led to considerations limiting the application of the Principle of Comprehension, according to which whenever there exist entities *xs*, there exists a class to which the *xs* belong as members. One way (adopted by Russell) to effect such limitations was by means of introducing a theory of types. Another (adopted by Zermelo and Fraenkel) was to construct an *axiomatic* set theory, where only particular cases of the Comprehension Principle were assumed.⁵⁷

On the other hand, reflection on these difficulties led Stanislaw Lesniewski to develop a theory which did away with classes-as-extensions, or *distributive* classes, as he called them, and made recourse only to Frege's other type of class, which he called *collective* classes.⁵⁸ He named his theory of collective classes *mereology*, the etymology of the term expressing the theory's interpretation as a theory of parts and wholes ('*meros*' is the Greek for 'part'). In what follows, I shall adopt Lesniewski's terminology, speaking of Frege's distinction as the distinction between distributive and collective classes.

⁵⁶ To take them as reflexive relations is to assume that an "improper part" is a part, and that an "improper subclass" is a subclass.

⁵⁷ See Machover 1996, 13-14.

⁵⁸ See Küng 1963, 105-115; Simons 1987, 101-104.

From the perspective of the common origin of modern theories of classes (i.e. theories of distributive classes) and of parts and wholes (i.e. theories of collective classes), it is understandable that Lesniewski did not consider mereology any more ontologically presumptuous than the class theories of Cantor, of Frege, and of Russell and Whitehead. Where the latter theories assume the existence of an entity (i.e. a class) which has certain individuals as members, mereology assumes the existence of an entity (i.e. a mereological sum) which has those individuals as parts. That is, both theories assume the existence of a single comprising entity, and they only differ with respect to the formal characteristics of the relations between the comprising entity and the individuals it comprises.

Moreover, the fact that class theory could only be safeguarded against contradiction by apparently placing limitations to the Comprehension Principle which have often been viewed as *ad hoc*, whereas mereology could be proven to be consistent without recourse to such limitations,⁵⁹ seemed to show that mereological comprising entities were if anything more rather than less acceptable than class-theoretical ones. It might be surprising, therefore, that the existence of mereological sums has generally been treated with more suspicion than the existence of sets or classes.

The explanation for this suspicion seems to lie in the fact that the distinction between distributive and collective classes came to be generally perceived as a distinction between abstract and concrete entities, respectively.⁶⁰ Our identificational intuitions seem to be led into conflict when we consider (alleged) arbitrarily specified concrete comprising entities, in a way that they do not seem to be with regard to corresponding abstract comprising entities.

If we assume, for example, that there exists a concrete entity which comprises a car and a hat on the other side of town, we feel unclear about the circumstances under which we should judge that this alleged comprising entity ceases to exist, in a way that we do not either with regard to the car or with regard to the hat. If 90% of the hat has been destroyed by fire, it might be clear to us that the hat has ceased

⁵⁹ See Simons p. 110, n. 23.

⁶⁰ We shall presently find that there are good reasons for doubting that distributive classes need necessarily be assumed to be abstract. However, it is likely that concrete distributive classes would be met with suspicions similar to those associated with concrete collective classes.

to exist. Has the alleged comprising entity ceased to exist as well? Classical mereology maintains that a sum of the *xs* exists if and only if all of the *xs* exist. According to this principle, the sum of the car and the hat has ceased to exist. Our intuitions, however, have very little to say as to whether a concrete entity which comprised the car and the hat has ceased to exist. It would seem no less plausible to claim that a concrete entity which comprised the car and the hat has continued to exist. Since our identificational intuitions are on unfirm grounds with regard to such alleged comprising entities, it is natural to doubt their existence.

The claim that mereology is no less justified in assuming with respect to any individuals *xs* that an entity exists which has them as parts than set-theory is in assuming that an entity exists which has those individuals as members is emphasized by Nelson Goodman in 'A World of Individuals'.⁶¹ He explains there that he uses 'individual' in a technical sense. Various associations arguably conveyed by the term in its ordinary use, such as, for instance, that of indivisibility, are explicitly excluded from Goodman's usage. Using 'individual' in this technical sense is, he says, as unobjectionable as using 'class' in the technical sense required in mathematical and logical contexts. In such contexts it is usually supposed that there exists a class whose only members are, say, the planet Mars, the tree next to me, and Henry's right hand, even though common discourse would arguably reject the existence of a class in such a case. Similarly, says Goodman, (classical) mereology assumes with equal justification that there exists an individual which is the sum of the same individuals (i.e. the planet, the tree and the hand).

It should be noted, however, that the analogy between mereology and set-theory is imperfect. It may be admitted that 'individual' is not less justifiably used as a technical term than 'class' is. However, set-theory does not presuppose that the entities comprised by a class are themselves classes, whereas mereology does presuppose that the entities comprised by an individual are themselves individuals. Thus mereology carries the implication that the comprising entity belongs to the same ontological category as do the comprised entities, an implication that is surely of more than mere terminological significance.

⁶¹ Goodman 1956.

Section 2.2

Distributive Classes and Concreteness

2.2.1 *The Question in Principle*

I have mentioned, in the former Subsection, that contemporary conceptions of wholes can be viewed as descendants of Frege's classes-as-collective-wholes, and so as versions of the notion of a collective class. This historical fact might incline one to assume dogmatically that concrete wholes (a notion which I will be looking at in detail in the next chapter) are necessarily to be conceived of as collective classes. Since the central proposal of the present work is to conceive of concrete wholes in a way which renders them *distinct* from any version of a collective class, it is important for my purposes to undermine any such dogmatic inclination.

With this in mind, it will be instructive to see that it is far from clear that concrete wholes cannot be conceived of as *distributive* classes, rather than as collective classes. Not that I will be urging the conception of concrete wholes as distributive classes. The notion of a Unity (not to be confused with 'unity')⁶² which I introduce in Part III, is neither a version of the notion of a collective class, nor one of the notion of a distributive class. However, by recognizing that it would not be obviously implausible to propose that some concrete wholes are distributive classes rather than collective classes, one becomes accustomed to the idea that although recent history of theory of wholes might suggest to the contrary, the notion of a concrete whole need not necessarily be understood in terms of the notion of a collective class.

It is not a difference in principle regarding ways in which a distributive class and a collective class can be one, which has led to the view that concrete wholes are collective classes. There seems to be no reason why a class of either type cannot be either atomic or non-atomic, or unified or ununified (Frege's distinction makes no assumption regarding the number of entities comprised by a class of either type, nor regarding any relations between the comprised entities). Classes of either type

⁶² On my use of 'unity' and 'Unity', see footnote in 1.1.3.3.

are linguistically one (for a class, whether distributive or collective, is one class), and are single entities (for a class of either type is generally spoken of using verbs in the singular).⁶³ Finally, it is not clear why it should be impossible for a distributive class to be non-monadic, if it is possible for a collective class to be - unless it is assumed that a distributive class (whose members are concrete) must be an abstract entity. I return presently to the question, whether indeed a distributive class must be abstract.

It might be thought that concrete wholes must be conceived of as collective classes rather than distributive classes, because the relation of a part to a whole is transitive, which fits *is a part of* (as characterised in connection with the notion of a collective class) but not *is a member of* (as characterised in connection with the notion of a distributive class). However, it may be doubted whether transitivity is indeed an essential feature of the relation *is a part of*. It has been argued, for example, that a certain nucleus is not a part of the heart, even though the nucleus is a part of a certain cell, and the cell is a part of the heart. And similarly, it has been argued that a handle is not a part of the house, even though the handle is a part of the door and the door is a part of the house.⁶⁴ If that is so, perhaps such wholes are better conceived of as distributive classes, allowing the relation between the cell and the heart to be considered *not* transitive.

Examining the ways in which classes of the respective types may be one, and the question whether the relation *is a part of* is transitive or not, does not seem to provide us with compelling reasons for thinking that a concrete whole must be viewed as a collective class and not as a distributive class. A more forceful consideration, however, is associated with the *concreteness* of concrete wholes. For there seem to be reasons to think that while collective classes may, in principle, be

⁶³ In Lesniewski's theory, a distributive class is conceived of as what I have called a 'multiple' entity.

Lesniewski argues that expressions used in his system which are apparently used to refer to distributive classes should be interpreted as 'shared names', making definite reference to the many class members, rather than to the class. That which is referred to by means of a shared name is what I have called a 'multiple entity'.

Lesniewski, however, took his account to constitute the claim that distributive classes *do not exist*. He clearly equated the view that a distributive class exists with the view that a distributive class is a *single* entity. See Küng 1963, 108-115.

⁶⁴ The argument regarding the example of the heart is found in Rescher 1955; that regarding the example of the house is found in Cruse 1979. Doubts regarding the transitivity of *is a part of* can also be found in Lowe 1989, 95 (footnote 9).

concrete, distributive classes may not. If that is so, then obviously a concrete whole cannot be a distributive class.

A terminological clarification is called for, before discussing this issue. I confine the term 'class' (on its own, i.e. not in combination with 'distributive' or 'collective') to entities studied in any familiar type of set theory.⁶⁵ In as much as the set theoretical classes share the essential features of distributive classes, I shall consider them as distributive classes of particular types. Any of the familiar types of set theory, however, specifies features of sets which are not among those which I noted above as distinguishing a distributive class from a collective class (for example, the assumption that a null class exists). Consequently, although I take it that all classes are distributive classes, I do not assume that all distributive classes are classes.

Now, classes are generally considered to be abstract entities. Since ^{such} entities are what most naturally springs to mind as falling under the concept of a distributive class, it is natural to suppose that abstractness is an essential feature of distributive classes. As for collective classes (in cases where such entities comprise concrete individuals), it is clear that from the outset they were assumed by theorists to be concrete entities.⁶⁶ Let us take this traditional view regarding collective classes for granted. Let us question, however, whether indeed distributive classes are essentially abstract.

There seem to be two main reasons for thinking that classes are abstract.⁶⁷ One is that it is assumed in set theory that there exists a null class, a class, that is, which has no members. The other is that it is assumed in set theory that a singleton (i.e. a class which has one and only one member) is not identical to its only member.

It may seem that if a class has no members, then it must be conceived of either as an abstract entity, or perhaps not even as an abstract entity but rather, simply, as

⁶⁵ Note that in some types of set theory the term 'set' is used to refer to all such classes. In set theories of other types a distinction is drawn, among entities designated by 'class' in this wide sense, between entities designated by 'set' and entities designated by 'class' in a narrower sense. See, e.g. Machover 1996, 16.

⁶⁶ For example, Frege assumes that a wood is a collective class of many trees; see Frege 1895, 87. For Lesniewski's similar view, see Küng 1963, 105f.

⁶⁷ See, for example, Russell 1919, 183; Grossmann 1983, 209-10, 214-216.

a fictitious entity.⁶⁸ As for the second point, it also seems to lead to the view that classes are abstract, because if the singleton of a pebble is an entity distinct from the pebble, there seems to be no concrete entity which is distinct from the pebble which is a plausible candidate for the role of singleton of the pebble.

Without further argument these two points are certainly not compelling. If it is possible that there should exist concrete atomic entities,⁶⁹ then it is not clear why a class which has no members should necessarily be abstract.⁷⁰ And if it is plausible to claim that a statue is different from the lump of clay which constitutes it, and yet to claim that both the lump and the statue are concrete entities, it may seem plausible to attempt to conceive of the singleton of a pebble, for instance, as a concrete entity which occupies the same place at the same time as the pebble, without being identical to the pebble. However, let us grant, for the sake of argument, that the two points can be developed convincingly, or that other satisfactory arguments can be produced, to show that classes are indeed abstract entities.⁷¹

What is important to see is that even if we grant that abstractness is an essential feature of classes (that is, of set-theoretical entities), it by no means follows that it is an essential feature of other types of entity falling under the general notion of a distributive class. The arguments for the abstractness of classes, which we have noted above, depend on the assumptions made in set theory, that null classes and singletons exist. These assumptions, however, depend on the acceptance of specific basic assumptions (expressed as unlimited or limited Comprehension Principles) regarding types of conditions under which it may be assumed that classes exist. If

⁶⁸ Grossmann, for example, claims that the empty class is a convenient fiction, and that this was also the position of the originator of set-theory, Cantor, himself. See Grossmann 1983, 214-15.

⁶⁹ For a discussion of the existence of such entities, see Simons 1987, 41-43.

⁷⁰ Indeed, David Lewis proposes to conceive of the null class as concrete, by identifying it with the mereological sum of all individuals, which is itself a concrete individual. See Lewis 1991, 10-15.

⁷¹ Grossmann provides two further reasons for thinking that classes are abstract (see Grossmann 1983, 209-16). I do not think his reasons are convincing, but there is no need to take issue with him on these points in the present context.

these assumptions were appropriately varied, the existence of the empty class and of singletons would not be guaranteed.⁷²

To illustrate that distributive classes might be assumed to exist which are not susceptible to the arguments which lead us to think that classes (that is, set-theoretical classes) are abstract, let us consider a "reduced" set theory as follows. According to this theory, the relations *is a member of* and *is a sub-class of* are assumed to have the same formal properties which they are assumed to have in familiar set theory. However, the existence of only a very limited range of classes is admitted in the reduced theory, as entailed by the following assumptions (which replace the various comprehension principles accepted in familiar set theories):

- A1 for all y , y is a class only if y is not an individual
- A2 for all y , y is a class iff for some xs , y is a class whose members are all and only the xs .
- A3 for any xs , for some y , y is a class whose members are all and only the xs iff the xs meet the following conditions:
 - (1) each of the xs are individuals, and
 - (2) for some u and for some v , u is one of the xs and v is one of the xs and $u \neq v$

According to this theory, if we assumed that there were three pebbles, a , b and c , and that these were the only individuals in the world, then there would be the following four classes (and only these): $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and $\{a, b, c\}$.

The assumptions rule out the existence of a null class and of singletons.⁷³ Therefore, the two main considerations for thinking that classes are abstract do not apply with regard to such a reduced theory of distributive classes.

⁷² In the case of Zermelo-Fraenkel set theory, for example, the existence of a *class* with a single member is guaranteed by the assumption that for any "definite" property P there exists a class $A = \{x: Px\}$. The existence, regarding any individual or set a , of a *set* whose only member is a , is guaranteed by the latter assumption together with the Axiom of Pairing and several definitions. These assumptions also guarantee the existence of an empty class and of an empty set. If we accepted only the existence of classes corresponding to *some* "definite" properties P , those consequences would not follow. See Machover 1996, 15-18.

⁷³ Assumption A2 rules out the null class. A3 rules out singletons. A1 and A3 rule out such classes as $\{a, \{a, b, c\}\}$. If such a class is admitted, and assumed to be concrete, then presumably it occupies precisely the same region of space, at the same time, as the class $\{a$,

I do not mean to suggest, of course, that this reduced set-theory is an attractive alternative to the more familiar set-theory, in as much as we seek in set-theory an adequate foundation for mathematics. What I am suggesting, rather, is that set-theoretical classes are not the only type of distributive class that we might consider, and that some of these other types are not implausibly assumed to be concrete entities.

2.2.2 *Russell's Aggregates as Concrete Distributive Classes*

Evidence that distributive classes have not always been conceived of as abstract entities can be found in Bertrand Russell's discussion of classes and aggregates in *The Principles of Mathematics*. For his account seems to imply that what he calls 'aggregates' are concrete distributive classes.

In the chapter 'Whole and Part',⁷⁴ Russell distinguishes between two kinds of whole: aggregates and unities. We need only consider aggregates at present. An aggregate is a class "regarded as the whole composed of all the terms in those cases where there is such a whole".⁷⁵ In other words, the aggregate is what Russell describes as a "class as one". In his account of the notion of a class, he argues that a distinction must be drawn between the notion of a class as many and the notion of a class as one. When we speak of the soldiers (sc. all the soldiers belonging to a certain army), for example, we speak of the class of soldiers as many. When we speak, on the other hand, of the army, we speak of the class of soldiers as one.⁷⁶ The existence of a class as one is not guaranteed in every case in which there exists a class as many.⁷⁷ However, in cases where the class as one exists, it may be

b, c}. Resistance to the possibility of being in the same place at the same time might encourage someone faced with such classes to prefer the assumption that they are abstract.

⁷⁴ Russell 1903, 137ff.

⁷⁵ Russell 1903, 139.

⁷⁶ See Russell 1903, 68-9.

⁷⁷ See Russell 1903, 68-9. Conditions under which the existence of a class as one must be denied are conditions which lead to contradictions such as the one known as Russell's Paradox. They are discussed in the chapter 'The Contradiction', Russell 1903, 101ff.

regarded as a "whole composed of all the terms", a whole of a certain sort which he calls 'aggregate'.⁷⁸

That an aggregate of concrete individuals is itself concrete is suggested by Russell's claim that a class as one is an entity of the same (logical) type as its members. He explains:

A class as one, we shall say, is an object of the same *type* as its terms; i.e. any propositional function $\phi(x)$ which is significant when one of the terms is substituted for x is also significant when the class as one is substituted.⁷⁹

Thus, for example, if 'x moves' is a propositional function which is significant when the value of x is one of the members of the class (e.g. one of the soldiers), then it is also significant when the value of x is the class as one (i.e. the army). If it is assumed (plausibly) that whenever something neither moves nor is at rest, 'x moves' is not significant when that thing is the value of x , it follows that the army either moves or is at rest. On these assumptions, the army, and in general any class as one (and thus a Russellian aggregate) whose members are concrete entities, is itself a concrete entity, according to the criterion I offered above for concreteness (see 1.1.3.1).

That an aggregate of concrete individuals is regarded by Russell as a distributive class can be seen from the way he emphasizes that aggregates involve two distinct relations, each of which can be loosely described as a relation between part and whole. These relations correspond precisely to the relations *is a member of* and *is a subclass of*, and therefore Russell's aggregates comply with Frege's criterion for distributive classes. For Russell contrasts the relation that "holds only between the aggregate and the single terms ... composing the aggregate", on the one hand, with "the relation to our aggregate of aggregates containing some but not all the terms of our aggregate" on the other hand:

For example, the relation of the Greek nation to the human race is different from that of Socrates to the human race;⁸⁰

⁷⁸ Russell 1903, 138-9.

⁷⁹ Russell 1903, 104-5.

⁸⁰ Russell 1903, 139.

Russell is assuming that the human race is an aggregate, and that entities that it contains may have two different types of relations to it: the relation which Socrates has to it (i.e. *is a member of*), and the relation which another aggregate, the Greek nation, has to it (i.e. *is a subclass of*).

It transpires then that Russell's aggregates are Fregean distributive classes, and yet that they are nonetheless concrete entities.

Section 2.3

Collections

In the present Section I wish to bring out a notion which underlies a certain familiar, non-technical use of the term 'collection'. On this basis I introduce a particular technical use of the term, which will be of service in later chapters, since it will transpire that wholes according to one conception are collections in this sense, while according to another conception they are not. And so this notion of a collection will provide a perspective from which to illuminate contrasts between different conceptions of wholes. In addition, a version of this notion of a collection will feature in the theory of Unities which I present in Part III, alongside the notion of a Unity itself.

It is of interest to discuss collections at this early stage, before we begin to discuss the notions of part and whole in detail, because the notion of a collection seems to be the source of a certain constraint which has been accepted - mistakenly in my view - as constitutive of the notion of a whole. The reason why this constraint seems to have been accepted, I suggest, is that wholes have tended to be conceived of as collections, in a broad sense which I explain below. In as much as to conceive of whole in this way involves an explication of the notion of a whole, at least in some respect, *in terms of* the notion of a collection, it seems appropriate to discuss collections prior to a discussion of wholes. In one important respect, the present work can be taken as an exploration of the possibility of elaborating a conception wholes according to which this constraint is not accepted.

Collections, I assume, are entities which can be referred to by means of expressions of the form 'the collection ...', such as 'the collection of pebbles', 'the collection comprising these paintings', etc. I assume that if *y* is a collection, then for some *xs*, *y* comprises the *xs*, so as to make it appropriate to say that the *xs* form *y*. I shall take the claim that *y* is a collection of the *xs* to be equivalent to the claim that *y* is a collection formed by the *xs*. Each of the *xs*, in such a case, is said to be a *member* of *y*.⁸¹

⁸¹ It is not assumed, however, that the relation designated by 'is a member of' in connection with collections is the same as the relation designated by the same term in connection with classes.

I assume further that in some contexts it might be said that a collection of meteorites is drifting in space, and is about to impact a planet; or that a colourful collection of pebbles is found on the beach. Collections spoken of in this way are taken to be entities that are linguistically one, single, and concrete. For 'collection' is used here in combination with a singular indefinite article 'a' (indicating linguistic oneness), with verbs in the singular, such as 'is' (indicating singleness); and collections are said to have such properties, or relations to other things, as *drifting*, *impacting*, *being found in* (indicating concreteness).

I conclude that the notion of a collection which is associated with one of the familiar uses of the term 'collection' is the notion of an entity which is concrete (if the comprised entities are concrete), linguistically one, and single.⁸² To say this much about the notion is, I take it, uncontroversial. By contrast, it is far less easy to say which constraints (if any) follow from this notion of a collection as regards atomicity, unity or monadicity, and also as regards being a distributive class or a collective class. Instead of investigating the issue of these further features at this point, however, I propose to take the notion of a collection as indeterminate in these respects, and to consider further determinations of the notion as corresponding to particular *species* of collections.

Thus there is, for instance, a species of collections which are essentially non-atomic (i.e., there are no *atomic* collections of this species). And there is a species of which being ununified, being non-monadic, and being a distributive class are essential features. At this stage we are investigating the generic notion, and we need not be concerned with the question whether collections belonging to any particularly determined species of this notion actually, or even possibly, exist.

At present, therefore, in speaking of a collection I am speaking of a concrete comprising entity, which is single and linguistically one, an entity which may or may not be atomic, unified or monadic.⁸³ A certain additional constraint does seem to follow, however, from the way 'collection' is used in the contexts I have in mind.

⁸² This is not to deny that there are other uses of 'collection' according to which a collection may lack one or more of these features.

⁸³ Notwithstanding the indeterminacy of the notion in these respects, according to the account I have offered so far, it is already sufficiently determinate in order for it to be contrasted with the notion Russell took 'collection' to connote in Russell 1903, 69.

By assuming as he does that a collection is referred to by means of an expression of the form ' A_1 and A_2 and ... and A_n ', Russell fails to guarantee (as I do by means of my

Suppose at t we find pebbles a, b, c, d and e at some distance from one another on an otherwise pebble-less beach. Suppose we assume that a, b, c and d form (at t) a collection, which we call C_1 . Suppose we also assume that a, b, c, d , and e form (at t) a collection, which we call C_2 . I submit that it is true on purely conceptual grounds, given these assumptions, that $C_1 \neq C_2$. If someone agreed with us in assuming that collections C_1 and C_2 are identified by such descriptions, and yet claimed that it is possible for C_1 to be identical to C_2 , we should say that that person has failed to grasp the concept of a collection and the associated concept of forming,⁸⁴ which we have applied in this case.

The qualifying remark 'at t ' is inserted in brackets, to allow for the possibility that a collection should gain or lose members in time. Ignoring this possibility for a moment, the point just made regarding collections formed by the pebbles may be generalized by saying that collections comply with a principle of Weak Extensionality, defined as follows:

Weak Extensionality

WE for all u , for all v , for any xs , for any ys

if (1) u comprises the xs , and v comprises the ys

and (2) each of the xs and each of the ys is an F (where ' F ' is a sortal term)
then

u is identical to v only if the xs are (severally) identical to the ys .

This principle rules out the possibility that a five pebble collection should be identical to a four pebble collection (assuming that it is impossible for a collection to gain or lose members). It does not, however, rule out the possibility that a five pebble collection should be identical, for example, to the collection formed by all the molecules comprised by the five pebbles. A principle which would rule this latter possibility out could be arrived at by dropping condition (2). I call this stronger

assumptions) that a collection is linguistically one, and that it is a single entity (in the sense explained in Subsections 1.2.2 and 1.2.7). I grant, however, that this assumption of Russell's is not *incompatible* with the assumption that a collection is single. Furthermore, in as far as he assumes that a collection is a class as one, he would seem to be assuming that it is a distributive class, rather than a collective class. For as I explained in Subsection 2.2.2, he seems to assume that a class as one is a distributive class.

⁸⁴ I will assume below that grasp of the relevant concept of a collection includes (and entails) grasp of the relevant concept of forming.

principle - one which I do not assume collections comply with - Strong Extensionality:⁸⁵

Strong Extensionality

SE for all u , for all v , for any xs , for any ys
if u comprises the xs , and v comprises the ys
then
 u is identical to v only if the xs are (severally) identical to the ys .

Principles WE and SE, as stated above, are meant to hold (when they do) irrespective of the time at which u comprises the xs or v comprises the ys . WE would clearly fail for collections if it were assumed that, for example, a collection of four pebbles can acquire a fifth pebble as a member. To accommodate this assumption, we require a temporally relativized version of WE, involving a temporally modified notion of the comprising relation (i.e. the notion of a relation designated by a temporally modified predicate 'comprises at t '): ⁸⁶

Weak Extensionality - relative to time

WET for all u , for all v , for any xs , for any ys , for all t ,
if (1) u comprises the xs at t , v is a collection of the ys at t
and (2) each of the xs and each of the ys is an F (where ' F ' is a sortal term)
then
 u is identical to v only if the xs are (severally) identical to the ys

Similarly, a temporally modified version of SE can be stated as follows:

⁸⁵ Strong Extensionality is very similar to the familiar principle known as the principle of Extensionality which is generally assumed to hold with regard to *classes*. According to Extensionality, if u is a class of the xs , and v is a class of the ys , then u is identical to v if and only if the xs are (severally) identical to the ys . Strong Extensionality differs from Extensionality only by replacing the biconditional 'if and only if' with the conditional 'only if'.

⁸⁶ I understand the relation between temporally unmodified notions and temporally modified ones in the way explained by Simons (1987, 176-8).

Strong Extensionality - relative to time

SET for all u , for all v , for any xs , for any ys , for all t ,
if u comprises the xs at t , and v comprises the ys at t ,
then
 u is identical to v only if the xs are (severally) identical to the ys .

For the sake of simplicity I shall conduct the discussion in terms of temporally unmodified notions. It will be a straightforward matter to adapt the conclusions to temporally modified terms.

To summarize, then: WE is a principle which may be said to be a feature of the concept of a collection, in that the acceptance of WE (for collections) follows necessarily from a grasp of the concept of a collection. The stronger principle, SE, is not in this sense a feature of the concept of a collection.

It seems clear, however, that a principle stronger than WE (though weaker than SE) can be formulated which, again, is a feature of the concept of a collection. Consider again the collection formed by pebbles a, b, c and d , i.e. C_1 . Now consider a collection formed by a, b, c, d and a certain molecule m drifting in the air in the vicinity of those pebbles.⁸⁷ Call this collection ' C_3 '. It would seem that admitting the distinctness of C_1 and C_3 follows from a grasp of the notion of a collection no less than admitting the distinctness of C_1 and C_3 does.

To formulate a principle from which the distinctness of C_1 and C_3 follows, I need to explain first what I mean by saying that some entities, the xs , are *coextensive* with some entities, the ys . The term 'coextensive' is chosen because one of the implications of the xs being coextensive with the ys is that of their collectively "extending" over the same region of space as the ys . Noting that for x to overlap y is for x to share a part with y ,⁸⁸ I define as follows:

⁸⁷ Perhaps it would be easier to visualize these entities as forming a collection if it is imagined that they are all drifting in proximity to one another in a region of outer space which is otherwise completely empty.

⁸⁸ See below, Item 3.1.4.2.

D1 the xs are coextensive with the $ys =_{\text{def.}}$

for all z , z overlaps one of the xs iff z overlaps one of the ys ⁸⁹

It can now be proposed that according to the stronger principle we were looking for, which governs the identity of collections, if u is a collection of the xs and v is a collection of the ys then u is identical to v only if the xs are coextensive with the ys . I am proposing, therefore, that a principle which may be called Coextensive Determination is a feature of the concept of a collection:

Coextensive Determination

CD for all u , for all v , for any xs , for any ys

if u comprises the xs and v comprises the ys

then

u is identical to v only if the xs are coextensive with the ys

The distinctness of collections C_1 (formed by four pebbles) and C_3 (formed by four pebbles and a molecule) from the example mentioned above clearly follows from CD. Molecule m overlaps one of the members of C_3 (indeed, it is one the members of C_3), but overlaps none of the members of C_1 . Therefore, C_1 and C_3 are not coextensive, and so according to CD they cannot be identical to one another.

Again, a temporally relativized version of the principle can be formulated. In addition to a temporally modified notion of the relation *comprises*, we require now a temporally modified notion the relation *are coextensive with*. This, in turn, requires a temporally modified notion of the relations *overlaps*, which can be defined in terms of a temporally modified notion of the relation *is a part of*. Thus we may say that x overlaps y at t iff x and y share a part at t . And the xs are coextensive with the ys at t iff for all z , z overlaps one of the xs at t iff z overlaps one of the ys at t . We then have:

⁸⁹ Generally in what follows, the expression 'one of the xs ' may be assumed to paraphrase 'at least one of the xs ' unless it is absolutely clear from the context that 'exactly one of the xs ' is meant.

Coextensive Determination - relative to time

CDT for all u , for all v , for any xs , for any ys , for all t ,
if u comprises the xs at t and v comprises the ys at t
then
 u is identical to v only if the xs are coextensive with the ys at t

I shall be assuming, then, that collections comply with principle CD (in case they are supposed to have fixed membership) or with CDT (in case they are supposed to have variable membership). If a type of comprising entities (e.g. collections) is governed
either by CD or by CDT then I shall say that an entity of this type has the feature of being 'coextensively determined'. Collections, therefore, are coextensively determined.

Collections, therefore, can be characterised as entities that are linguistically one, single, concrete, and coextensively determined. Both concrete distributive classes, and concrete collective classes, it seems, must be considered to be collections in this sense.

For, in as much as distributive classes are generally assumed to comply with Strong Extensionality, and compliance with Strong Extensionality implies compliance with Coextensive Determination (as can be readily seen on inspection), we conclude that *concrete* distributive classes belong to a type of collections.

As for concrete collective classes, we shall see - at least in as much as they are fairly interpreted in terms of the notion of a mereological sum - that they comply with either CD or CDT, and so they too count as collections.

Chapter 3

Theory and Pre-theory of Wholes

Section 3.1

Pre-theoretical Conception of Wholes

3.1.1 *The Contrast between Pre-theoretical and Theoretical Conceptions*

In discussing the relations between parts and wholes, we must distinguish between pre-theoretical and theoretical conceptions. Consider, for comparison, a similar distinction that might be drawn in discussing the nature of heat. We have, on the one hand, a pre-theoretical understanding of the notion of *being hot*, an understanding which determines the appropriate use of the term 'hot'. On the other hand, we have theoretical proposals which are intended to explain what *being hot* consists in, such as, for example, the (no longer plausible) proposal that *being hot* consists in the presence of a type of material substance known as 'caloric', or the alternative proposal that *being hot* consists in the molecules making up the hot thing being in such a state as to possess a mean kinetic energy the degree of which is higher than some specified standard. Some of the main tasks of a study of the nature of heat are the introduction and development of, and decision between, such alternative proposals.

The case is similar with regard to discussing the nature of wholes and parts and the relations between them. Theoretical proposals are put forward with the intention of illuminating a subject matter which is identified by our pre-theoretical understanding of the terms 'whole' and 'part'. The field of such theoretical study is called 'mereology'.

The analogy between the theory of heat and mereology applies notwithstanding the fact that the former is an empirical theory and the latter is a

metaphysical theory. In both cases it is a crucial question to what extent the theory is successful in providing an explanatory account of our pre-theoretical notions. It is important to keep this in mind, because one sometimes encounters an attitude towards mereology as if it has made precise, once and for all, our conceptions of whole and part.⁹⁰ This is misleading in two ways. First of all, it fails to give sufficient due to the variety of mereological theories that have been proposed, and to the difficulties involved in deciding between them. Secondly, it fails to note that although the range of theories that have been proposed is wide, it is still demonstrably not exhaustive.

3.1.2 *The Broader Pre-theoretical Use of 'whole' and 'part'*

Pre-theoretically we speak of a great variety of entities as wholes, or as parts. Indeed, there is a broad sense of the term 'whole' according to which it seems to have an almost universal application. For example, we may claim to be considering the building as a whole, or the whole building. And then it seems acceptable to say that the building *is* a whole, although such a claim is admittedly more typical of philosophical than common discourse. Similarly the city, the earth, or the universe, can be considered as wholes. Furthermore, it seems to be equally acceptable to speak of the whole tennis match, the whole symphony, the whole proposition, the whole story, or the whole theory. Turning to 'part', we find that it has a similarly wide application, again, if it is taken in a sufficiently broad sense. The building, we may say, is a part of the city, the earth is a part of the universe, the proposition is a part of the theory, the movement is a part of the symphony, and the symphony is a part of the opus of that composer.

With respect to these very general senses of 'whole' and 'part', the chief restriction to their application seems to derive from the way these terms are often taken as *correlative* to one another. To say of *x* that it is a part is somewhat like saying of *u* that she is a lover: In both cases a relational attribute is involved. Just as the latter implies that *u* is a lover of *something*, so the former implies that *x* is a part

⁹⁰ For example, in Lewis 1986a.

of something. Generally, the claim that x is a part tends to be understood as implying that there is some y of which x is a part. Correlatively, this y is considered as a whole with respect to x , i.e. a whole one of whose parts is x . Similarly, saying that y is a whole is understood as implying that there is some x which is a part of y , and this x can be described as a part with respect to which y is a whole.

The correlativity of 'whole' and 'part' leads to a restriction of the application of these terms, if we make the further assumption that an entity x cannot be a part of itself - that is, that the relation *is a part of* is irreflexive. In common discourse it seems that 'part' is often understood in such a way as to imply that *is a part of* is indeed irreflexive.⁹¹ If *is a part of* is irreflexive, then if we say that x is a part we imply that for some y , y is not identical to x , and y is a whole with respect to x . This would suggest, for instance, that 'part' does not apply to the universe (if indeed there is an entity which is appropriately referred to by means of the expression 'the universe'), assuming that no entity which is distinct from the universe could plausibly be held to have the universe as a part. Similarly, if *is a part of* is irreflexive, then by saying that y is a whole we imply that for some x , x is not identical to y , and x is a part of y . On this assumption it would seem that 'whole' does not apply to a geometrical point, for example. For there seems to be no entity distinct from the point which could plausibly be taken to be a part of the point.

In relevant theoretical contexts it is commonly assumed that *is a part of* (by contrast with *is a proper part of*, of which we say more in what follows) is actually a reflexive relation. According to this assumption, the application of 'whole' and 'part' is even less restricted.

We note, then, in the context of the pre-theoretical use of both 'whole' and 'part', a broad sense which is associated with a very wide-ranging application. It is noteworthy that in this sense even classes can be considered as wholes - and this not only with respect to their sub-classes but also with respect to their members. As Rolf Eberle says:

One speaks of parts and wholes with respect to many kinds of relations. For example, a concrete individual may be regarded as the whole of its qualitative

⁹¹ Frege seems to take the assumption (by Schröder) that *is a part of* is reflexive to be an unobjectionable *extension* of a more familiar, irreflexive notion. See Frege 1895, 87.

parts, of its physical molecules, or of the events in its history. A set $\{A, B\}$ is a whole composed of the parts $\{A\}$ and $\{B\}$ relative to inclusion, but a whole of the parts A and B with respect to the ancestral of membership.⁹²

Indeed, Cantor's famous definition of 'set' has sets described as wholes:

By a 'set' we understand any collection M of definite well-distinguished objects m of our perception or our thought (which are called the 'elements' of M) into a whole.⁹³

Similarly, Russell considered the term 'whole' as appropriate for describing the class-as-one, taking either its members or its subclasses as parts, emphasizing, though, that the notion of a whole associated with the former assumption is distinct from that associated with the latter assumption.⁹⁴ It may be concluded that the terms 'whole' and 'parts', taken in their widest sense, can be used to describe any comprising entity and the entities it comprises, respectively.

3.1.3 *The Narrower Pre-theoretical Use of 'whole' and 'part'*

A narrower pre-theoretical use of 'whole' and 'part' is one which takes *concrete* entities as its paradigmatic cases. A stone, a tree, a church, a horse, are all concrete entities, and are all described as wholes (or parts) in this narrower sense. The correlativity of 'whole' and 'part' is a feature of this use as it is of the broader use described above. Thus to say of a tree that it is a whole implies that there is some (concrete entity) x which is a part of the tree - for example, one of its branches. And to say of a stone that it is a part implies that there is some (concrete entity) x of which the stone is a part - say the castle's wall. A *non-concrete* entity might be said to be a whole in this narrower sense only if the relations between it and its parts (i.e. those entities that are said to be parts correlative to its being a whole) are taken

⁹² Eberle 1970, 32.

⁹³ Cantor 1895, 481, quoted by Black 1971.

⁹⁴ See Russell 1903, 138-9. See also discussion above, Subsection 2.2.2.

to be the same as the relations that are taken to hold between a concrete whole and its parts.⁹⁵

It is with this narrower pre-theoretical sense of 'whole' and 'part' that the discussion of wholes and parts is usually assumed to be concerned. This explains why the theory of classes is not generally thought to be a part of mereology.⁹⁶ The theory of classes is not thought to be offering theoretical accounts of concrete entities, and in particular not of wholes in the narrower sense. Again, in terms introduced in the former chapter, 'whole' and 'part' in the narrower sense may be generally used to describe a concrete comprising entity and the entities it comprises, respectively. In what follows I shall be assuming, in speaking about wholes and parts, this narrower pre-theoretical usage.

3.1.4 *Pre-theoretical Terminology*

3.1.4.1 *'Part' and 'Whole'*

Since different theoretical conceptions of wholes are going to be considered, each associated with its own terminology, it will be useful to make a few comments with a view to clarifying and disambiguating our talk about wholes and parts prior to commitment to one or another theoretical conception.

One may begin by assuming that some entities (in what follows, unless explicitly stated otherwise, I use 'entity' to speak of *concrete* entities) stand in the relation *is a part of* to other entities. If x is a part of y , then it is said that y is a *whole* with respect to x . Thus 'is a whole with respect to' may be taken to designate the converse of the relation *is a part of*. The claim that y is a whole with respect to x is

⁹⁵ Thus Lewis (1991, e.g. 3ff.) argues in that classes are wholes in the narrower sense (in particular, that they are mereological sums) with respect to the singletons of their members. This claim must be distinguished from the weaker and much less controversial claim of Eberle's, noted above (in connection with Eberle 1970, 32), that a class can be considered to be a whole, in the *broader* sense of the term, with respect to its members.

⁹⁶ Even Lewis, who thinks that mereology *applies* to classes, takes set-theory to involve a non-mereological relation, namely the relation between an individual and its singleton. See, for example, Lewis 1991, 87.

alternatively expressed by saying that *y* has *x* as a part. And the claim that for some *x*, *y* is a whole with respect to *x*, can be expressed by saying simply that *y* has parts.

Note that all of this is consistent with saying that *y* is a whole with respect to many entities (which are not identical with one another), and with saying that *x* is a part of many entities (which are not identical with one another).

Suppose we have five letter-shaped pieces of plastic, 'P', 'l', 'a', 't' and 'o' (they have been named after the letter types of which they are tokens, respectively). These plastic pieces may be spoken of obliquely as 'letters'. Suppose these letters are arranged so that a token of the name 'Plato' exists, and let this token be referred to, simply, as 'Plato'. Assume that both 'Plato' and each of the letters are entities.

In this case, 'P' stands in the relation *is a part of* to 'Plato', so that 'P' is a part of 'Plato'. Correlatively, 'Plato' stands to 'P' in the relation *is a whole with respect to*, so that 'Plato' is a whole with respect to 'P' (or 'Plato' has 'P' as a part). Indeed, 'Plato' has parts.

3.1.4.2 'Overlap' and 'Disjoint'

If two entities have a part in common then they are said to *overlap* one another. That is, if *z* is a part of *x*, and *z* is a part of *y*, then *x* and *y* overlap one another (this could also be expressed by saying that *x* overlaps *y*). If entities do not overlap one another then we say they are *disjoint* from one another.

Suppose 'Pla' and 'ato' are entities which are parts of 'Plato', where 'Pla' has 'P', 'l' and 'a', but neither 't' nor 'o', as parts; and where 'ato' has 'a', 't' and 'o', but neither 'P' nor 'l' as parts. Thus 'Pla' overlaps 'ato', and it similarly overlaps 'P', or 'l'. 'P', however, is disjoint from 'ato', and similarly it is disjoint from 'l', or 'o'.

3.1.4.3 'Make up'

We say that the letters 'P', 'l', 'a', 't' and 'o' *make up* 'Plato'. The predicate 'make up' must be understood here as applying collectively, not distributively, to all the letters. The way it applies is therefore analogous to the way 'weigh a thousand pounds' applies, in a given case, to seven people, where it is understood that none of those people weighs a thousand pounds, but that their weights added up come to one thousand pounds.

If each of the *xs* is a letter, and the *xs* are properly among 'P', 'l', 'a', 't' and 'o', then the *xs* do not make up 'Plato' (thus, for example, 'l', 'a', and 'o' do not make up 'Plato'). Moreover, if each of the *ys* is a letter, and 'P', 'l', 'a', 't' and 'o' are properly among the *ys*, then the *ys* do not make up 'Plato'.

In the theories I shall be calling 'classical mereology' and 'neoclassical mereology', *make up* is taken to be the same as the relation *compose*, defined as follows (note that I am not taking 'compose' to be a pre-theoretical term):⁹⁷

D1 the *xs* compose *y* =_{def.}

for all *z*, the *z* overlaps one of the *xs* iff *z* overlaps *y*

We shall see that this is the relation the *xs* bear to *y* if, and only if, *y* is a sum of the *xs*. In what I call the 'theory of Unities', however, *make up* is not understood in terms of *compose* in this way.

3.1.4.4 'The Parts'

On the basis of the assumptions that have already been explicitly made regarding the case of 'Plato' and its parts, it is clear that 'P', 'l', 'a', 't' and 'o' are not the only parts of 'Plato'. For it has been assumed, for example, that 'Pla' and 'ato' are parts of 'Plato'. And of course it is natural to assume also that each of the letters is itself a whole with respect to various chunks of plastic. If 'P', for example, was

⁹⁷ Van Inwagen assumes (in 1990, Section 2) that 'the *xs* compose *y*' is not equivalent to '*y* is a sum of the *xs*'. By contrast, Lewis takes these to be equivalent, as can be seen from his account in 1991, 73-4.

broken into two pieces, we are very likely to say that each of these pieces was formerly a part of 'P'.

Nevertheless, it is convenient and natural in many contexts to speak of 'P', 'l', 'a', 't' and 'o' as *the* parts of 'Plato'. This may be described as a qualified, or context-bound, use of the expression 'the parts'. For often, in considering wholes and parts, one takes the following perspective. One considers that there are many (i.e. two or more) entities, the *xs*, ignoring any parts that they may have. And one considers further that there is an entity, *y*, which the *xs* make up, ignoring any other parts which *y* might have. Since only the *xs* have fallen under consideration so far as parts of *y*, and since they are *all* the parts which have fallen under consideration, it is appropriate in the context to describe them as *the* parts of *y*.

In the discussion that follows it will be assumed that 'the parts' is used to refer (simultaneously) to all the entities that are being considered *in that context* as making up the relevant whole. Thus we may freely speak, for example, either of many cells, or of many molecules, as *the* parts of a human organism. Context will determine which of these two options, or indeed of these and many more conceivable options (e.g. many nuclear particles, or many chunks of flesh and bones, etc.), is the intended one. And if this is not explicitly specified by context, we may still use the expression 'the parts' as referring, indifferently, either to the many molecules, or to the many cells, or to any other plausible candidates for consideration as being comprised by the human organism.

To agree on such a use of 'the parts' is not to rule out, however, the possibility that in some cases 'the parts', or 'all the parts', is used to refer *strictly speaking* to all the parts of the whole under consideration. Used in this way (say, in connection with 'Plato'), one refers by means of 'the parts' to the *xs*, such that *any* entity that is a part of 'Plato' is among the *xs* (including, for example, any molecule that is a part of one of the letters). It will be clear from the context which of these different uses is intended.

3.1.4.5 'Proper Part' and 'Improper Part'

A distinction is usually made between two types of parts: parts that are *proper*, and parts that are *improper*. The pre-theoretical notion of a *proper* part can be ambiguously explained as that of a part which does not include everything included by the whole. An *improper* part is a part which is not a proper part. Thus 'P', 'Pla' and 'Plat' are proper parts of 'Plato'. While if 'Plato' is at all a part of 'Plato', it must be an improper part. The notions of a proper part and an improper part can be made more precise (in a theoretical context), controversially, in terms of the following assumption:⁹⁸

- A1 for all x , for all y , if x is a proper part of y then
 y has a proper part which is disjoint from x

Alternatively, it might be proposed that the claim that a proper part "does not include everything included by the whole" should be rather understood in terms of the following different assumption:

- A2 for all x , for all y , if x is a proper part of y then
 y has a part which is not a part of x

Given the assumptions (widely held in mereology) that every entity is a part of itself, but no entity is a proper part of itself, it can be seen that it is consistent with A2, but not with A1, that (for some x and some y) x is the *only* proper part of y .⁹⁹

It is clear from the foregoing paragraphs that the notion of an improper part is understood as that of a part which *does* "include everything included by the whole". It is usually assumed that this implies that an improper part of a whole is necessarily *identical* to the whole. This assumption makes it possible to define 'is a part of' in terms of 'is a proper part of' and 'is identical', in a way which (as we shall see) is characteristic of classical mereology:¹⁰⁰

⁹⁸ Simons refers to this assumption regarding the relation *is a proper part of* as the Weak Supplementation Principle. See Simons 1987, 28.

⁹⁹ Simons argues that the assumption that an entity might be the *only* proper part of a whole is unacceptable on fundamental, conceptual grounds. I do not find his position on this matter compelling; however, I will not take issue with it in the present work. See Simons 1987, 26.

¹⁰⁰ See Simons 1987, 11, 26, 37 (D2 is equivalent to Simons's SD1).

D2 x is a part of $y =_{\text{def.}}$

x is a proper part of y or x is identical to y

It will be seen (in Subsection 4.3), however, that in neoclassical mereology D2 is rejected, together with the assumption that an improper part of a whole is identical to a whole.

3.1.4.6 'Atom'

If a concrete entity has no proper parts, it is said to be an *atom*. I shall assume that atoms not only do not have proper parts, but are such that if any entities, the x s, are comprised by an atom y , then *each* of the x s is identical to y . Thus I am assuming that atoms are atomic entities, in the sense of 'atomic' explained in Subsection 1.2.3.

Atoms, as mentioned here, must not be confused of course with microscopic entities of a certain type which are known by the same name and typically studied in physics. The "atoms" of physical theory are thought to have ^{proper} parts (e.g. protons, neutrons, electrons), and to that extent are not atoms in our sense. Note also that an entity's being indivisible (in the sense that it is physically impossible to break it up, to cause parts of it to separate and disperse), does not entail its being an atom in our sense. For it may well be that some individuals have parts which it is physically impossible to separate.

Section 3.2

Preliminaries to the Theoretical Conception of Wholes

3.2.1 *Broad Outline of Alternative Theoretical Conceptions of Wholes*

In Chapter 2 it was noted that two conceptions of a comprising entity emerged towards the end of the 19th Century: that of a distributive class, and that of a collective class. As these conceptions developed, the assumption established itself that entities of the former type were essentially abstract entities, while entities of the latter type, in cases in which they comprise concrete entities, are themselves concrete. Since the pre-theoretical notion of a whole (we speak here of wholes in the narrower sense) is the notion of a concrete comprising entity, it is not surprising that the notion of a collective class became the chief theoretical model according to which wholes were conceived.

It is appropriate to describe the notion of a collective class as offering a theoretical conception of wholes, because it entails claims about wholes and their relations to individuals which are their parts which do not seem to follow incontrovertibly from the rather unspecific pre-theoretical beliefs we have about wholes. For example, if wholes are conceived of as collective classes, then the relation *is a part of* is understood as a transitive relation. Pre-theoretically, however, it seems by no means incoherent to propose that the relation *is a part of* is not transitive.¹⁰¹

Indeed, it was seen in Chapter 2 (see Section 2.2) that there are reasons for thinking that distributive classes are not essentially abstract (although, admittedly, set-theoretical classes are plausibly conceived of as abstract entities). If distributive classes of some specified type are concrete entities, then it would seem that the notion of a distributive class offers an alternative theoretical model for wholes. If a whole whose parts are the *xs* is conceivable either as a collective class of the *xs* or as a distributive class of the *xs*, then the notions of a collective class and of a distributive class are alternative theoretical conceptions of the whole, providing alternative accounts of the nature of that whole. For the case here seems similar to

¹⁰¹ On this matter, see Subsection 2.2.1 above.

that briefly described in Subsection 3.1.1, involving the theory of heat, where two alternatives were available for accounting for the nature of heat, one in terms of the theory of caloric, the other in terms of the kinetic theory.

Modern mereology, as we shall see, has by and large been a theory of collective classes. Within the framework of mereological discussions, collective classes are more commonly known as sums. Notwithstanding its varied development and the many forms it has taken, current mereological thinking does in general take core features of the notion of a sum as its foundation. It follows that the received mereological wisdom, in the main, urges the theoretical conception of *wholes as sums*.

According to the conception of wholes as sums, one replaces the pre-theoretical terms 'whole', 'part' and 'make up' with the theoretical, or technical, terms 'sum', 'part' and 'compose', where the assumptions laid down by the theory, using the latter terms, give those terms their specific technical meanings. (Note that 'part' is homonymous; it appears both among the pre-theoretical terms - in a pre-theoretical sense, and among the theoretical terms - in a distinct, theoretical sense).¹⁰²

Turning to the alternative available model for conceiving of whole^s, namely, the notion of a distributive class, if we call a concrete distributive class 'Class' for convenience (with upper case 'C'), then this alternative can be described as the conception of *wholes as Classes*.

According to this conception, one reads for the pre-theoretical 'whole', 'part' and 'make up' the theoretical 'Class', 'member' and 'form', respectively. Again, the meanings of the latter terms are determined by their roles in stating the assumptions of the theory.

This alternative has generally received very little attention, exceptions to this neglect being, arguably, Russell's theory of wholes in *The Principles of Mathematics*, and Simons's theory of so called "concrete classes" in *Parts*.¹⁰³ Even if it is granted

¹⁰² These senses should not be confused with one another. Indeed, in as much as different versions of the conception of wholes as sums involve different assumptions with regard to the formal properties of *is a part of*, they may be said to understand 'part' in a different way from one another.

¹⁰³ See Russell 1903, 138-9; Simons 1987, 144ff. I discussed Russell's wholes in Subsection 2.2.2.

that both speak of concrete distributive classes, neither of the authors has argued that the conception of wholes as Classes is in any respect preferable to their conception as sums, at least as regards the type of wholes that are generally at the centre of interest of mereological discussions, i.e. as regards entities which I have described as *prima facie* individuals. The case has not been made that entities such as a man, or a table, or a planet, should be conceived of as Classes rather than as sums.

The notion of a sum, by contrast, has received much attention and has been widely applied in theorizing about *prima facie* individuals. This wide exposure, together with the apparent absence of an attractive alternative, perhaps explains why it is sometimes forgotten that the conception of wholes as sums, however plausible, is in principle one which might be rejected, or at least one which might in principle be deemed inappropriate in connection with particular types of cases.

Against this background, what I shall be calling the Theory of Unities is to be seen in broad brush-strokes as offering a distinct third model, alongside the two currently available ones of wholes as sums and wholes as Classes. As far as I have been able to ascertain, this third model is completely without modern precedent. It may be described as ^{the}conception of *wholes as Unities* (the reason for choosing the term 'Unity' will become clear as we progress).

Although the conception of wholes as Classes is not without interest, it will not be treated in the present work. My main interest lies in the conception of wholes as Unities. Most of the discussion that follows is aimed at a comparison between the predominant conception of wholes as sums, and their less familiar conception as Unities. Much of the present Part is devoted to a presentation and discussion of the conception of wholes as sums. Among other points, I will be emphasizing the flexibility of this conception, expressed in its ability to accommodate both "lower grade" notions of wholes, such as those often associated with the terms 'heap' or 'aggregate', and a range of traditional "higher grade" notions of wholes such as that of a Hegelian organic whole, that of a Gestalt-theoretical whole, and that of a Husserlian dependent whole.

Notwithstanding this flexibility, however, I hope to show that the conception of wholes as Unities is not only possessed of an intrinsic interest and plausibility,

but also has in certain respects tangible advantages over their conception as sums. Part III will be devoted, in the main, to this conception of wholes as Unities.

According to the conception of wholes as Unities, one interprets the pre-theoretical terms ‘whole’, ‘part’ and ‘make up’ (in some contexts, at least) as the theoretical terms ‘Unity’, ‘element’ and ‘underlie’ respectively.

My comments on the different terms used in connection with the three models are summarily presented in the following table:

Model	Comprised Entities	Comprising Relation	Comprising Entity
wholes as Classes	members	form	Class
wholes as sums	parts	compose	sum
wholes as Unities	elements	underlie	Unity

The differences between the models are not merely terminological, of course. I shall examine them in some detail in Part III. It will be seen, for example (to give a quick initial impression of the substantial differences between the models) that in the case of Classes the *xs* may form the same entity as the *ys* only if the *xs* are (severally) identical to the *ys*. In the case of sums, by contrast, the *xs* may compose the same entity as the *ys* only if the *xs* are coextensive with the *ys*. Although Classes and sums differ in this way, however, they are both *coextensively determined* comprising entities, and so they both count as collections of certain types (See Section 2.3 above). Whereas in the case of Unities, the *xs* may underlie the same entity as the *ys* even if the *xs* are neither (severally) identical to the *ys* nor coextensive with the *ys*, and indeed, Unities are not coextensively determined comprising entities, and as a consequence, they are not collections.

3.2.2 Another Look at Frege's Distinction

It was noted in Chapter 2 that a distinction between alternative conceptions of a comprising entity was drawn by Frege in 'A Critical Elucidation of some Points in E. Schröder's *Vorlesungen über die Algebra der Logik*'. Let us look more closely at his distinction. Suppose the *xs* are pebbles, and *u* is a collective class of the *xs*, and *v* is a distributive class of the pebbles. Let us say that each of the pebbles is a *component* of *u*, and similarly that each of them is a component of *v*. Let us say that a collective class of the *ys*, where the *ys* are among the *xs*, is a *sub-class* of *u*. And similarly, a distributive class of the *ys*, where the *ys* are among the *xs*, is a sub-class of *v*.

Frege pointed out that the two types of comprising entity must be distinguished from one another, because although entities of both types may be said to "include" both their components and their sub-classes, in distributive classes these two relations of "inclusion" are distinct, whereas in collective classes they are not. That is, the relation which a component of a collective class bears to the collective class has the same formal characteristics as the relation which a sub-class of a collective class bears to the collective class. Whereas the relation which a component of a distributive class bears to the distributive class has different formal characteristics from those of the relation which a sub-class of a distributive class bears to the distributive class.

It seemed to Frege (for reasons that will be considered presently) appropriate to identify the relation of a component to a collective class as the relation *is a part of*. I use 'is a member of' to express the relation Frege takes to hold between a component of a distributive class and the distributive class.¹⁰⁴

Thus if *u* is a collective class of pebbles, then every component of *u* (i.e. every pebble) and every sub-class of *u* (i.e. every class the components of which are among the pebbles) is a part of *u*. On the other hand, if *u* is a distributive class of pebbles, then every component of *u* is a member of *u*. As regards sub-classes of distributive classes, Frege assumed that the relation each of them bears to the class is precisely the relation *is a part of*.

¹⁰⁴ Frege's own term for 'is a member of' in Frege 1895 is 'subter'.

We have, then, a formal distinction between two relations: the relation *is a part of*, and the relation *is a member of*. Entities that are “included” in a collective class all bear the former relation to the collective class. As for items that are “included” in a distributive class, some bear the former relation to the distributive class, others bear the latter relation to it.

The formal features by which Frege distinguishes the relation *is a part of* from the relation *is a member of* are expressed by the following two axioms:

A1 Every entity is included in itself.

A2 If one entity is included in a second, and this in a third, then the first is likewise included in the third.¹⁰⁵

Making appropriate substitutions,¹⁰⁶ these axioms apply to the relation *is a part of*, that is, in the case of a collective class, both to the relation between a component and the collective class and to the relation between a sub-class and the collective class. They equally apply in the case of a distributive class to the relation between a sub-class and the distributive class. Importantly, however, the axioms do not apply in the case of a distributive class, to the relation between a component and the distributive class.

We have then the following two axioms as (at least partially) characterizing the relation *is a part of* according to Frege:

A1' Every entity is a part of itself.

A2' If one entity is a part of a second, and this of a third, then the first is likewise a part of the third.

Axiom A1' expresses the claim that *is a part of* is a reflexive relation, and A2' expresses the claim that it is a transitive relation.

The view that axioms A1' and A2' apply to a relation which is appropriately described as the relation *is a part of* was apparently suggested to Frege by examples in which the formal characteristics of this relation are evidently realized, examples which it was natural to judge as paradigmatic examples exhibiting the relations

¹⁰⁵ See Frege 1895, 86. Frege uses 'domain' instead of 'entity' at this point of his discussion, adapting to the terminology used in the relevant study of Schröder's.

¹⁰⁶ That is, substituting 'is a part of', 'is a class-member of' or 'is a sub-class of' for 'is included in', as the respective cases require.

between parts and wholes. One such example that he offers is that of the areas into which a plane surface is divided by two (non-parallel) sets of parallel lines. Any two pairs chosen respectively from these sets delimit an area in the shape of a parallelogram. Clearly, some such areas are *contained* (in the spatial sense) in some others. Given our understanding of the notion of spatial containment, it would seem undeniable that axioms A1 and A2 apply (on appropriate substitutions) in the case of this example with respect to the relation *is contained in*, and this relation in turn is naturally described as a relation between a part of an area and an area of which it is a part. Thus it is plausible to take these axioms as capturing a characteristic feature of the relation *is a part of*.

3.2.3 Discussion of Formal Features of the Fregean Conception

The assumption that axioms A1' and A2' apply to the relation *is a part of* is a natural one, and one that is very widely held. Eberle, for instance, characterizes the most general notion of a relation which is to qualify *prima facie* as a species of *is a part of* by saying that a whole, in the most general sense,

may be identified with the supremum of a set relative to an arbitrary partial ordering.¹⁰⁷

A relation is a partial ordering if and only if it is reflexive, antisymmetric and transitive.¹⁰⁸ To say that a whole is the *supremum* of a set relative to some partial ordering is to imply that the relation *is a part of* is a partial ordering, and thus, in

¹⁰⁷ Eberle 1970, 32. Eberle uses here the term 'sum', but as his foregoing remarks make plain, he takes 'whole' and 'sum' to be synonymous. I use the term 'sum' only for the more specialized theoretical notion characteristic of classical mereology and of theories which follow with regard to certain general principles, as will be explained in what follows.

¹⁰⁸ These features are defined as follows (see Eberle pp. 13-14):

The *field* of a relation R is the set of all items x such that either x bears R to something or something bears R to x . Suppose F is the field of R . Then:

A relation R is *reflexive* if for all x in F , xRx .

A relation R is *irreflexive* if for all x in F , not xRx .

A relation R is *symmetric* if for all x, y in F , if xRy then yRx .

A relation R is *asymmetric* if for all x, y in F , if xRy then not yRx .

A relation R is *antisymmetric* if for all x, y in F , if xRy and yRx then $x=y$.

A relation R is *transitive* if for all x, y, z in F , if xRy and yRz then xRz .

particular, that it is reflexive and transitive.¹⁰⁹ Therefore, according to Eberle, reflexivity and transitivity are features of the relation *is a part of*, taken in its most general sense.

Similarly, Simons states that the principles embodying the claim that the relation *is a proper part of* is a *strict partial ordering*¹¹⁰ are

partly constitutive of 'part', which means that anyone who seriously disagrees with them has failed to understand the word.¹¹¹

And again, summarizing at the end of his study, he claims that being a strict partial ordering is part of

the minimum we can require of a relation if it is to be one of proper part to whole.¹¹²

Given the definitional connections between the relations *is a proper part of* and *is a part of*, to which Simons subscribes,¹¹³ this means that he agrees with Eberle (and with Frege) in taking the relation *is a part of*, in its most general sense, to be both reflexive and transitive.

We have noted already, however, that doubts can be, and have been, raised as to the assumption that *is a part of* is transitive (and similar doubts are applicable regarding the assumption that *is a proper part of* is transitive). Thus it is not completely clear that A1' and A2' provide us even with *necessary* conditions for a relation to be a relation of the type *is a part of*.

It is clear, however, that even if A1' and A2' correctly characterize such necessary conditions for the relation *is a part of*, they do *not* provide sufficient

¹⁰⁹ Eberle defines 'supremum' as follows:

$\text{sup}_R S$ [the *supremum*, relative to R , of S] = the unique object y which is such that

- (1) every member of S bears R to y
- (2) for all z , if every member of S bears R to z , then y itself bears R to z
- (3) R is a partial ordering

We say that $\text{sup}_R S$ *exists* just in case there is one and only one object y such that the conditions (1), (2) and (3) are satisfied.

¹¹⁰ A relation R is a *strict partial ordering* if it is irreflexive, antisymmetric and transitive (see Eberle 1970, 14)

¹¹¹ Simons 1987, 11.

¹¹² Simons 1987, 362.

¹¹³ In fact, Simons considers two notions of *is a part of*, as defined by reference to that of *is a proper part of* (the latter is taken as primitive in his system). See Simons 1987, 26 (SD1) and 112 (SD15).

conditions for a relation of this type, for reflexivity and transitivity characterize many relations that are clearly not species of *is a part of*. For example, the relation *is lighter than or equal in weight to* is reflexive and transitive, but the claim that stone *A* is lighter than or equal in weight to stone *B* is obviously compatible with the claim that stone *A* is *not* a part of stone *B*.

This is not to say, however, that the axioms are not sufficient for Frege's purpose of distinguishing between two distinct relations of inclusion, and establishing that classes, as required for the purposes of the foundations of mathematics, must involve a relation of inclusion that does not fulfil these axioms (namely, the relation *is a member of*). Indeed, the specification of axioms for the relation *is a part of* is only incidental to this purpose of Frege's, as he was not interested (in the context of the 1895 article) in developing a theoretical account of wholes.

By contrast, classical mereology, versions of which were first introduced by Lesniewski as Mereology and by Leonard and Goodman as the Calculus of Individuals, was explicitly intended by its authors as a theoretical account of wholes. Dissatisfaction with various aspects of set-theory led these authors to seek to develop a theory based on an alternative notion of a comprising entity, that of a collective class.

One of the main reasons why it was reasonable to believe that there existed entities which could be described as collective classes was that the characteristics which collective classes were held to have seemed to accord well with our beliefs about entities of a very familiar type, namely, wholes (in the narrower sense). The interpretation of the notion of a collective class as a theoretically clarified conception of a whole endowed the notion with a certain plausibility which allowed it to be considered as a serious alternative to the notion of a distributive class, once it was realized that this latter notion was in many ways problematic.

Thus the authors of classical mereology aimed to provide a theoretical account of the notion of a whole, with the expectation that such a theoretical account, conceived as an elaboration of the notion of a collective class, could replace the problematic notion of a distributive class (as developed in set-theory) for a variety of philosophical purposes.

Classical mereology speaks, in a given case, of a comprising entity such that any of the comprised entities is related to the comprising entity by the relation *is a part of*, where this relation is taken to be both reflexive and transitive, as in Frege's account. Thus classical mereology speaks of comprising entities which are understood as collective classes. As we shall see, however, classical mereology has more to say about the way such comprising individuals are to be characterized.

II Sums

Chapter 4

Classical and Neoclassical Mereology

Section 4.1

Principles of Classical and Neoclassical Mereology

4.1.1 *The Conception of Wholes as Sums*

An important motivation for setting up the system of classical mereology is illuminatingly explained by Nelson Goodman.¹ He begins by pointing out that the relation *is a member of*, on which the theory of classes is founded, can be described as a *generating* relation. Accepting the theory of classes involves the consequence that, if we assume that certain entities exist, we are committed to assuming that certain further entities exist. Thus if we assume that three trees exist, we are committed to assuming further that a class whose members are all and only those trees exists.

The qualms Goodman has about the theory of classes is not that it involves an extension, as such, of our ontological commitments. What he finds objectionable is

¹ See Goodman 1956.

that, firstly, it extends our ontological commitments in a way which violates a certain principle of parsimony, according to which, as he says, there should be no distinction without distinction in content. The notion of content relevant here is the notion of the ultimate entities which analysis reveals to be responsible for "generating" the entity under consideration. Suppose, for example, a , b and c are atoms. Consider a class A whose members are a and b , a class B whose members are a , b and c , and a class C whose members are A and c . According to the principle of extensionality applying to set-theory, classes A and C are distinct from one another, since they have different members. However, A and B are entities which Goodman would describe as having the same content. For they are both "generated" by precisely the same atoms, a , b and c .

Secondly, the ontological extension that set-theory involves is in his view extravagant and implausible, in that the assumption that an entity exists leads to the assumption that an *infinite* number of entities exist. For assuming that individual a exists, for instance, then according to set-theory a singleton of a exists, and this singleton is an entity which is not identical to a . And furthermore the singleton of a 's singleton exists, identical neither to a nor to its singleton; and a singleton of the second singleton exists, etc. Note that here too we have a violation of Goodman's principle according to which there can be no distinction without a distinction of content, for any one of the infinite hierarchy of singletons has precisely the same content as any other singleton of the hierarchy, i.e., a .

Classical mereology² or, as he calls it, the Calculus of Individuals, is a system which involves a generating relation (which he identifies with the relation *is a part of*), but one which does not lead to a violation of the principle of no distinction without distinction in content. According to classical mereology it is assumed that if there are many individuals³ then there is a *sum* which has those individuals as parts. The notion of a sum is as follows:

D1 y is a sum of the x s =_{def.}

z overlaps y if and only if z overlaps one of the x s.

² I use 'classical mereology' as short for what Simons calls 'Classical Extensional Mereology'. See Simons 1987, 37.

³ Note that in my own presentation of mereology, I shall use 'entity' instead of 'individual', for reasons explained in Item 1.1.3.3.

As we shall see below, the axioms and definitions of mereology guarantee that such sums comply with Goodman's principle of content.⁴ Furthermore, they guarantee the transitivity and reflexivity of the relation *is a part of*, and thus that mereological sums are collective classes.⁵

It was noted in Subsection 4.2.3 that the elaborated notion of a collective class, i.e., the notion of a sum, was interpreted by the original authors of classical mereology as a formally clarified conception of a whole. Thus classical mereology carried with it the proposal that wholes are to be conceived of as mereological sums. In other words, the proposal is to understand the claim that *y* is a whole which is made up of the *x*s as the claim that *y* is a sum of the *x*s, where this latter claim is taken in accordance with D1.

If 'sum' is used to refer not only to the temporally unmodified notion defined in D4, but also to the corresponding temporally modified notion (of which more below), it may be said that the assumption that wholes are to be conceived of as sums goes virtually unchallenged in modern discussions of wholes and parts. This is not to say that *other* assumptions made in classical mereology regarding sums are not challenged. Indeed, the rejection of several assumptions regarding sums leads one from classical to *neoclassical* mereology,⁶ and to a transformed notion of a sum. I describe this as a transition from the conception of a whole as a classical sum to the conception of a whole as a neoclassical sum. However, a neoclassical sum complies with D1 (or with a temporally relativised version of D1) no less than does a classical sum. To that extent, neoclassical mereology continues to uphold the conception of wholes as sums.

⁴ See principle P6 of the next Subsection.

⁵ The transitivity of *is a part of* follows from P1 and P4 (see next Subsection); the reflexivity of *is a part of* follows from P1 and the reflexivity of *is identical to*.

⁶ I use the term 'neoclassical mereology' to refer to the weaker theory which abandons one or more of the classical assumptions, as explained below.

4.1.2 *Sums in Classical Mereology*

Classical mereology is an axiomatic system: the theorems of classical mereology are all derivable from a specified set of axioms and definitions. As is often the case with such systems, different axiom bases, and correspondingly different sets of definitions, might be chosen, all of which are equivalent in being equally sufficient for deriving the same body of theorems which is classical mereology. An example of such differences between equivalent axiom bases can be found in comparing the system presented by Nelson Goodman with that presented by Simons.⁷ In Goodman's system, 'overlaps' is taken as primitive, and 'is identical to', 'is a part of' and 'is a proper part of' are defined in its terms, as well as terms designating other notions such as 'sum' and 'product'. In the system of Simons, by contrast, 'is a proper part of' and 'is identical to' are taken as primitive, and 'overlaps', 'is a part of', and other terms are defined in their terms.

In presenting the principles of classical mereology, my main aim is to arrive at a list which makes it possible to contrast classical and neoclassical systems in the most natural and perspicuous way. Simons's list of definitions and axioms provides a very helpful point of departure for developing such a list. However, one of the purposes guiding the construction of his list is that of bringing together all and only those definitions and axioms required to provide a basis from which the theorems of classical mereology can be derived. This is not my purpose here.

First of all, I may leave out definitions which introduce notions that we have already introduced as belonging to the pre-theoretical framework for the discussion of wholes and parts. Thus I leave out definitions introducing the notions of overlap and disjointness. Secondly, I leave out definitions of mereological terms which will not occur in our discussion, such as 'product', 'difference', 'Universe' and 'complement'. Thirdly, I add to Simons's axioms an assumption which is derivable from his definitions and axioms (and thus one which is not logically independent of the axioms and definitions), but needs to be added explicitly in some neoclassical theories which do not accept all the classical principles. Describing definitions and

⁷ See Goodman 1951, ch.2; Simons 1987, 25-41.

axioms generically as *principles*, the following list of principles of classical mereology is arrived at:⁸

Definition of 'part' in Terms of 'proper-part' and 'identical'

- P1 x is a part of $y =_{\text{def.}}$
 x is a proper-part of y or x is identical to y

Definition of 'sum'

- P2 y is a sum of the x s $=_{\text{def.}}$
for all z , z overlaps y iff z overlaps one of the x s⁹

Asymmetry of *is a proper part of*

- P3 for all x , and for all y , if x is a proper part of y
then y is not a proper part of x

Transitivity of *is a proper part of*

- P4 for all x , for all y , for all z ,
if x is a proper part of y and y is a proper part of z
then x is a proper part of z

Supplementation of Proper Parts

- P5 for all x , for all y , if x is a proper part of y
then for some z , z is a proper part of y and z is disjoint from x

Uniqueness of sums

- P6 for all u , for all v , for any x s, if u is a sum of the x s and v is a sum of the x s
then u is identical to v

Universal Existence of Sums

- P7 for any x s, for some y ,
 y is a sum of the x s

⁸ Note that P1 was already mentioned as D2 in 3.1.4.5.; P2 was mentioned above as D1 in 4.1.2.

⁹ By saying that z overlaps *one* of the x s it is meant that z overlaps *at least* one of the x s.

Principles P1 and P2 correspond to Simons's definitions SD1 and SD9, respectively. P3, P4, P5 and P7 correspond to Simons's axioms SA1, SA2, SA3 and SA24, respectively.¹⁰ The principle of uniqueness of sums, P6, is one which Simons takes to follow from his definitions and axioms.¹¹

¹⁰ See Simons 1987, 37.

¹¹ Principle P6 is explicitly stated by Lewis as one of "the basic axioms of mereology" (see Lewis 1991, 74). To see that Simons considers P6 to follow from his definitions and axioms, note that he provides on pages 38-41 a list of theorems which he takes to be derivable from the axioms and definition. Among these we find SCT16:

(1) $x = y$ iff for all z , z overlaps x iff z overlaps y [Simons's SCT16]

Now suppose

(2) u is a sum of the x s, and v is a sum of the x s [assumption]

Then it follows that

(3) for all w , w overlaps u iff w overlaps one of the x s [from P2]

Similarly,

(4) for all w , w overlaps v iff w overlaps one of the x s [from P2]

Therefore

(5) for all w , w overlaps u iff w overlaps v [from (3) and (4)]

Therefore,

(6) $x = y$ [from (1)]

Section 4.2

Limitations of the Notion of a Classical Sum

Classical mereology is a very "democratic" theory of wholes and parts, in that it requires that for any xs there is one and only one entity which is their sum. There are no particular conditions which have to be met by the xs in order for there to be a sum of the xs . The principle of Universal Existence of Sums (P7) guarantees that a proton tens of thousands of kilometres below the surface of the Sun, and the fly buzzing now in the room, for example, have a sum.

That is, there is an entity y of which one part is buzzing here peacefully and another part is embroiled in radically different circumstances, very far away, with vast heaving masses and great expanses of open space between them, masses and expanses which are not themselves parts of the same individual. When, in the course of time, the molecules which are now parts of the fly become widely dispersed as parts of the atmosphere, the earth, and many and varied animals and plants, an entity z which is the sum of the fly's present molecules and the proton far away in the sun will still exist (assuming that all those molecules and the distant proton still exist), and this sum is identical to y . The identity of z to y is guaranteed by the principle of Uniqueness of Sums (P6), given the identity of the entities of which both y and z are a sum.

4.2.1 *The Aggregate Argument*

Perhaps the most familiar objection to such a conception of wholes is that it is incompatible with the familiar observation that the existence of at least some types of entity depends on parts of those entities being related to one another in certain ways. This consideration dates back at least as far as Aristotle's Aggregate

Argument:¹² in the case of at least some types of entity, namely, an entity that is *prima facie* an individual, if the parts of the entity are sufficiently dislocated, separated from each other, disrupting certain spatial and functional relations between them, then those parts may all continue to exist, and yet the entity of which they are parts ceases to exist.

For example, we might imagine a tree all of whose molecules are separated from each other and dispersed over a wide region, as a consequence of certain physical forces coming to bear on the tree (e.g. such as would be engendered by a proximate nuclear explosion). Clearly it is plausible to claim that the tree ceases to exist as a result of this dispersal of its parts.

According to the classical mereological view, however, an entity is identified with the unique sum of its parts, and the sum exists if and only if the parts exist, so that the sum exists in cases of the type just mentioned both before and after the dislocation of the parts. It is a consequence of the classical mereological view, then, that an entity exists both before and after the dislocation of its parts, in conflict with the plausible point made by the Aggregate Argument.

Let us consider possible responses which a classical mereologist might make to the point made on the basis of the Aggregate Argument.

4.2.2 *The Four-dimensionalist Response*

A classical mereologist might deny the claim that entities may persist, strictly speaking, through time. Concrete entities must be conceived of according to this view as occurrents rather than continuants.¹³ If this is assumed, then nothing which exists strictly speaking at some time t can exist at some other time t' . In particular, the molecules of a tree which exists at t cannot be identical to certain widely dispersed molecules which exist at t' , and so the sum of the former cannot

¹² See Aristotle, *Metaphysics* 1041b11-31. The term 'Aggregate Argument' is used by Scaltsas to refer to this argument. See his discussion in Scaltsas 1994a, 64-5, and Scaltsas 1992, 196-8.

¹³ The contrast is briefly discussed above, in Item 1.1.3.1.

be identical to the sum of the latter, the classical mereological assumptions notwithstanding.

This widely held position is sometimes described as the 'four-dimensionalist view', since according^{to it} trees and other familiar entities are taken not as they seem to be - three-dimensional entities that persist through time - but rather as four-dimensional entities which have time as their fourth extensional dimension.

Four-dimensionalism, as we see, supports a classical mereological conception of wholes, and it is not surprising therefore to find typical classical mereologists such as Goodman and Lewis upholding four-dimensionalism.¹⁴ Adopting four-dimensionalism, however, does not provide the classical mereologist with a solution to a similar difficulty that might be raised in modal, instead of temporal terms. For consider an example in which the molecules of a tree *might be* (rather than *are*) dispersed at *t*. If the tree is identical to the sum, then the tree would exist even if the molecules were dispersed (for on classical mereological assumptions, the sum would exist in that circumstance). But clearly, the tree would not exist under those (counterfactual) conditions, and so the tree cannot be identical to the sum.

The appeal to four-dimensionalism does not resolve the difficulty in this case, because what is at issue is not properties which the tree and the sum have at different times, but rather modal properties, i.e. properties which they have at different possible worlds. To reply to this modal version of the Aggregate Argument, the classical mereologist would have to provide an account according to which it is denied that individuals have such modal properties.¹⁵

More importantly, four-dimensionalism involves a rejection of one of our most deeply rooted intuitions about the world and about ourselves, namely that some entities do not have a merely instantaneous existence, but persist in their existence over a period of time. The deep roots of these intuitions reveal themselves in the difficulties which are encountered in trying to spell out in detail how the world might be described in a *purely* four-dimensionalist language.¹⁶ A physicist who

¹⁴ See Goodman 1956; Lewis 1991 (e.g. 77). Note, however, that Whitehead according to Simons is an example of a four-dimensionalist who is not a classical mereologist, in that he denies the assumption of universal existence of sums (P7); see Simons 81-6.

¹⁵ This point, with regard to a similar problem, is made by Simons in Simons 1987, 122.

¹⁶ For a discussion of difficulties at making sense of a four-dimensionalist account, see Chisholm 1976, Appendix A; van Inwagen 1987, 36-7; Simons 1987, 122-127.

makes recourse to four-dimensionalism as providing a convenient framework for interpreting an abstractly formulated physical theory, may not need to worry about the availability of such a purely four-dimensionalist language. The ultimate untenability of four-dimensionalism would threaten not the *truth* of the physical theory (which is assessed on empirical grounds), but rather only the way in which the theory is interpreted. A metaphysician, by contrast, must be worried about this, for her interest focuses precisely on such empirically un-evaluable issues.

It will be seen in Part III that an argument involving mereological considerations, which has sometimes been taken as providing a compelling reason for embracing a four-dimensionalist view,¹⁷ is answerable by revising one's mereology rather than by revising one's conviction about the persistence of entities through time. In any case, however, I will not be pursuing further the question of four-dimensionalism further in context of the present work. Leaving it aside, I only wish to register my lack of sympathy for four-dimensionalism, and my assumption that it stands in favour of a theory if it allows us to avoid resorting to that view.

4.2.3 *An Alternative Classical-mereological Response*

A different defence of classical mereology, in the face of the Aggregate Argument, can be made without recourse to four-dimensionalism. According to this defence, no entity ceases to exist on the dispersal of the tree's molecules. Rather, there is a property, *is a tree*, which a sum of molecules may acquire or lose. At some point in the history of the sum^{it} turns into a tree, and at some later point it ceases to be a tree. Incidentally, it is likely to be not much later, given the speedy process of replacement of molecules which make up what we usually take to be one and the same tree. Suppose S_1 is a sum of molecules m_1, m_2, \dots and m_n , such that S_1 is a tree at t_1 . Suppose one of the molecules, m_n , is lost at t_2 , perhaps by becoming a part of an insect. Then at t_2 S_1 cannot be a tree (assuming that no part of a tree can

¹⁷ I have in mind Mark Heller's discussion (in Heller 1988) of the puzzle which I treat below in Section 9.3. A variety of views on the argument he is discussing is surveyed by Simons (1987, 117-121).

be a part of an insect), although S_2 , a sum of molecules m_1, m_2, \dots and m_{n-1} , may well be a tree at this time.

This view is somewhat similar to the one Frederick Doepke describes as the Reductivist View.¹⁸ This view rejects the arguments for distinguishing (for example) between a gold ring and the lump of gold which constitutes by claiming that strictly speaking, only the ultimate particles making up the ring should be held to exist. Gold rings, and similarly trees, are not real entities, but logical constructions out of real entities.¹⁹

The defence of classical mereology we are currently considering is not committed to a position quite as strong as the Reductivist View, for this defence acknowledges the existence not only of ultimate particles but also of arbitrarily specified sums of such particles. The classical mereologist can claim that there exists an entity (i.e. a sum of the relevant ultimate particles) which is a tree at t , or an entity which is green at t , claims that are not available to the Reductivist because none of the ultimate particles are either trees or green.

The classical mereologist who argues along the lines suggested, however, is nevertheless radically revisionary of common sense. For the argument requires us to deny that entities fall into the kinds recognised by common sense. The world, according to the view that transpires, does not contain entities which are appropriately described as trees, horses, mountains, buildings or chairs. The sum of such and such ultimate particles might be a tree at t_1 , a car at t_2 , a horse and a collection of three boulders at t_3 (or any one of indefinitely many other possibilities). Thus the sum is no more an entity of one of these kinds than of another, and since only ultimate particles and their sum exist, no entity which can be described as belonging to one of these kinds can be said to exist. The view implies, therefore, that only kinds of entities that might be acknowledged are those of the ultimate particles or of sums of ultimate particles.

¹⁸ See Doepke 1984, 57-60.

¹⁹ Doepke, as well as Simons (both following Wiggins), argue that the ring and the lump are distinct entities which occupy the same region of space at the same time. See Wiggins 1967, Doepke 1984, Simons 1987, 112-7, 210ff.

Chisholm's view is similar to the Reductivist View, in that he takes entities which are not susceptible to flux of parts to be *entia successiva*, where he understands the notion of such an entity to be the notion of a logical construction. See Chisholm 1976, ch.3 (note esp. 98); Appendix A.

The question as to whether such a view is ultimately tenable or not, I leave as lying outside the scope of the present inquiry. In any case it seems to me that a mereological theory that would force such a view on us, on purely *mereological* grounds, should not be eagerly accepted. At least not before the possibility of devising a metaphysically less limiting mereological theory is carefully looked into.

4.2.4 *Problems Associated with the Principle of Uniqueness of Sums (P6)*

The Aggregate Argument in the form presented above emphasizes problems associated with the principle of Universal Existence of Sums (P7). A similar argument may be presented, however, which seems to suggest problems with the principle of Uniqueness of Sums (P6).

A syllable (see next paragraph, regarding type/token ambiguity here), it is said, cannot be identified with the classical sum of the letters which compose it. For example, the syllable 'SO' cannot be identified with the classical sum of 'S' and 'O'. Not only is this because according to P7 a sum of the letters exists even in a case where the letters are not appropriately juxtaposed to one another, as, for example, in the case in which the letters are part of the expression 'das Obst' (just as much as the tree does not exist if the molecules are widely dispersed). The syllable 'SO' cannot be identified with the classical sum of the letters for the further reason that those letters compose more than one syllable, i.e. both 'SO' and 'OS'. Since the sum is unique, according to P6, and since 'SO' and 'OS' are different from one another, at most one of the syllables could be identified with the sum. But there would seem to be no more reason for identifying the sum with 'SO' than with 'OS'. Thus we should conclude that neither 'SO' nor 'OS' can be identified with the classical sum of 'S' and 'O'.²⁰

Thus formulated, however, the argument is misleadingly vague about whether it speaks of types or tokens of syllables and letters. Leaving aside relations between types as outside the domain of our discussion, it is difficult to present a case along

²⁰ The example is taken (but used to make a slightly different point) from John McDowell's discussion of Plato's *Theaetetus*, (1973, 241).

the lines of this argument for the existence of letter-tokens which have more than one sum at some time *t*. As Lewis shows in *Parts of Classes*, several possible clarified versions of this argument do not lead to a violation of the principle of Uniqueness of Sums (P6), if one is willing to pay the theoretical prices indicated in the foregoing paragraphs in connection with preserving the principle of universal existence of sums (P7), e.g. if one is willing to be committed to four-dimensionalism.²¹

A stronger case for the violation of the principle of Uniqueness of Sums, however, might be made with in connection with examples such as that of a clay statue and the lump of clay which constitutes it, or that of a person and the organism which constitutes her/him. A clay statue and the lump of clay which constitutes it are associated with different criteria of identity, respectively. Evidence for this is that, for example, the lump is judged to survive certain deformations which result in the destruction of the statue. Both the statue and the lump, however, would seem to be sums of the same clay particles (for something overlaps either of them iff it overlaps one of the clay particles), so it seems that wholes cannot be adequately accounted for as unique classical sums. Various responses to this argument are available to the classical mereologist, responses, however, which typically rely on controversial accounts of identity and of modality.²²

4.2.5 *Problems Associated with Mereological Constancy*

We have looked at problems which arise in connection with particular principles among those assumed in classical mereology, P6 and P7. A noteworthy difficulty with classical mereology, however, is associated with a feature of the theory that is not expressed in some particular principle, so much as in the use of *temporally unmodified* predicates in formulating the theory. This implies that the relation *is a proper part of* (as well as other mereological relations defined in its terms) is treated as if entities bear this relation to one another timelessly, in the way entities which are not thought of as being in space and time are taken to bear

²¹ See Lewis 1991, 78-9. See also a related discussion in Simons 1987, 113-4.

²² See for example Noonan 1993.

relations to one another (e.g. in the way the number 8 bears the relation *is greater than* to the number 5).

Typical concrete entities are normally assumed to be capable of losing or gaining parts. It is assumed to be possible, for example, for a molecule which is a part of a tree at t , not to be a part of the tree at t' (where $t \neq t'$). The fact that classical mereology makes use of temporally unmodified predicates, however, renders it impossible even to formulate such common sense claims in its terms. The terms of classical mereology are adequate, at best, for expressing claims about entities that are *mereologically constant*, that is, entities which have whatever parts they have as long as they exist. They are unsuitable for expressing claims about entities that are *mereologically varying*.

Indeed it is controversial whether the common sense assumption that entities may lose or gain parts is to be upheld in metaphysical theory. The assumption is clearly incompatible with four-dimensionalism, but it is rejected also by theorists who reject four-dimensionalism. One such theorist, Roderick Chisholm, for example, has argued for a thesis of 'mereological essentialism', according to which the parts of a entity - that is, of a *genuine* entity²³ - are essential to it. This implies, as he says, that "if y is ever part of x , y will be part of x as long as x exists."²⁴ Genuine entities, therefore, are mereologically constant, and an account of them does require temporally modified mereological predicates.

Even Chisholm, however, requires *some* notion of the relation *is a part of* which incorporates temporal modification (namely, the notion which applies to entities which are not genuine, i.e. to *entia successiva*).²⁵ And to the extent that one does not think it appropriate for four-dimensionalism, or some reductive view of the sort touched upon in 4.2.3, to be imposed on oneself merely as a consequence of the fact that the terms in which a theory is formulated are unsuitable for describing mereological variation, to that extent one is bound to be critical of classical mereology in this respect as well as those formerly discussed.

²³ Chisholm distinguishes between genuine entities, which are necessarily mereological constant, and so called '*entia successiva*', which may well be mereologically varying, but are reducible to genuine entities, and are to be viewed as logical construction upon genuine entities. See Chisholm 1976, 98; 154.

²⁴ Chisholm 1976, 145.

²⁵ See Chisholm 1976, 155.

This brief and selective sketch of problems met with by an adherent of classical mereology should suffice to indicate why its rejection might seem attractive to many authors. I am not interested within the present framework, however, to settle the issues which weigh for and against the acceptability of classical mereology as an account of wholes and parts, but rather to offer a degree of fleshing out of what the classical and neoclassical theories involve and how they contrast to one another. I wish to show that the contrast between them is compatible with their sharing certain features. These^{are} features which pertain to all theories which account for wholes as sums, whether classical or neoclassical. I shall presently be noting further problems with classical mereology which are associated with the general features of theories of wholes as sums.

Section 4.3

The flexibility of the Notion of a Neoclassical Sum

It is important to realize that a criticism of classical mereology is not necessarily a criticism of the conception of wholes as sums. The discussion of the last Subsection suggests that some of the most striking difficulties that arise in connection with classical mereology as a theory of wholes and parts are associated with two of the principles which it lays down: The principle of uniqueness of sums (P6), and the principle of universal existence of sums (P7). The notion of a sum as defined above, however, entails neither of these principles. Moreover, neither of these principles is entailed by the conjunction of principles P2-P5. Principles P2-P5 (or their temporally modified analogues - of which more below) may be taken, therefore, as providing the core for what might be described as '*neoclassical*' mereologies. Any mereology which is weaker than the classical one, asserting P2, P3, P4 and P5 (or their temporally modified analogues), but not all of the principles P1-P7, I shall designate accordingly as '*a neoclassical mereology*'.

As for the omission of P1 from the neoclassical core, the reason for this can be seen as follows.²⁶ By denying P6, one opens up the possibility that two distinct entities may be sums of precisely the same parts. For example, it becomes possible for a gold ring to be distinct from a lump of gold, and yet at the same time to be a sum of the same molecules. In that case the ring and the lump would be *superposed* with one another, for we define:

D1 u is superposed with $v =_{\text{def.}}$

for some xs , u is a sum of the xs and v is a sum of the xs

Moreover, it becomes possible for the ring to be distinct from the lump even if both entities have exactly the same proper parts. In that case the ring is not only superposed with the lump, but actually *coincides* with it, given the definition:

²⁶ I find Simons's explanation of reasons for omitting P1 somewhat unclear. See Simons 1987, 112; 180; and 210-11.

D2 u coincides with $v =_{\text{def.}}$

for all x , x is a proper part of u iff x is a proper part of v

The need to deny P1 arises, however, even if one is to justify the weaker claim that the ring is superposed with the lump. For it seems correct to say not only that the ring is a sum of the molecules, however, but also that it is a sum of the lump (for something overlaps the ring iff it overlaps the lump). And if the ring is a sum of the lump, then the lump is a part of the ring.²⁷ But the lump cannot be a *proper* part of the ring, according to principle of Supplementation of Proper Parts (P5), for the ring has no proper parts which are disjoint from the lump (the ring, recall, is assumed to be a sum of precisely the same molecules as the lump).

As long as P5 is maintained, therefore, it follows from the denial of P6 that an entity u may have an improper part v which is not identical to u . This, however, contradicts principle P1.

4.3.1 *Types of Neoclassical Sum*

It is an expression of the flexibility and versatility of the notion of a sum, that it is compatible both with the classical theory and with a variety of neoclassical theories. In virtue of this versatility, some of the most important phenomena associated with relations between wholes and their parts can be apparently satisfactorily accounted for in terms of the conception of wholes as sums. Under neoclassical assumptions, types of sums can be introduced, with features tailored to account for phenomena which were not accounted for satisfactorily in classical mereological terms.²⁸

²⁷ This follows from SCT31 and SCT34 in Simons 1987, 39.

²⁸ Note that Simons reserves 'sum' for what I have called 'classical sums'. I use the term 'sum' more generally, so that y is a sum of the x s if and only if it complies with the condition stated in definition D1 of 4.1.1 (with respect to those x s) - whether in temporally unmodified terms or in temporally modified terms.

One type is that of sums of which it is an essential characteristic that certain relations hold between their parts, or that the parts have certain properties.²⁹ I call sums of this type '*conditioned sums*'. By contrast, classical mereological sums can be described generally (but not without exceptions) as '*unconditioned sums*'.³⁰ To simplify the discussion, I will consider at present only conditioned sums which involve conditioning *relations* (rather than conditioning properties).

It is convenient to consider such conditioning relations under two headings. One is that of relations such that, if y is a sum of the x s, the existence of the x s does *not* depend on their having those relations to one another. For example, if the x s are small pebbles (less than an inch across), the relation of being three inches apart from each other is such a relation. Similarly, the relation of gravitationally attracting one another with a certain force f_0 (in Newtons) is a relation on which the existence of the pebbles does not depend. With certain paradigmatic examples of such relations in mind, examples in which the relations set up the spatial structure according to which the x s are arranged, sums which are conditioned by such relations can be called '*structured sums*'.

The other type of conditioning relations to consider are relations such that, if y is a sum of the x s, the existence of at least some of the x s *does* depend on the x s having those relations to one another. For example, Aristotle famously claims that if a finger is cut off from the hand of a living person, that finger ceases to exist.³¹ Therefore the existence of the finger depends, according to Aristotle, on its being appropriately attached to the living person's hand. Sums conditioned by such relations may be called '*internally dependent sums*'.

A second type of sums which the neoclassical framework can accommodate is that of sums which are not unique with respect to their parts. That is, under neoclassical assumptions, both y and z may be sums of the x s, and yet y is not

²⁹ I speak of relations as universals, or as types, and not as universal instances (tropes) or tokens.

³⁰ Regarding some relations, or some properties, the existence of the x s depends on their having those relations to one another, or those properties. Consequently, the existence of a sum of the x s depends, *even on classical assumptions*, on the x s having those relations to one another, or those properties.

³¹ Although a different entity, which looks much the same as the former finger, does now exist, and 'finger' is used homonymously to describe the attached entity and the detached one. See Aristotle *Metaphysics* 1035b23-25.

identical to z. Sums with this characteristic may be called '*non-unique sums*', in contrast with the '*unique sums*' of classical mereology.

Finally, a third type of sums which can be accommodated within the neoclassical framework is that of sums which have different parts at different times. Working with a temporally modified version of the relation *is a part of*, and treating it within a neoclassical framework (i.e. assuming appropriately adapted version of P2-P5, but not P1, P6 or P7), sums can be introduced which I describe as '*mereologically varying sums*', which are contrasted with the '*mereologically constant*' sums of classical mereology.

Summarizing the general types of sum which find their place in a neoclassical framework, but not in a classical one, we obtain the following picture:

- I conditioned sums
 - Ia structured sums
 - Ib internally dependent sums
- II non-unique sums
- III mereologically varying sums

By distinguishing these as different types of sums, I do not wish to suggest that if a sum is of one of these types it may not also be of one of the other types. On the contrary, it is quite possible that a neoclassical mereologist will combine several of these features in elaborating what he or she takes to be the notion of a sum which offers an adequate theoretical conception of wholes.

For example, a neoclassical theory capturing the views of Chisholm regarding wholes would be one which featured conditioned sums, which are at the same time unique and mereologically constant. The views of van Inwagen, on the other hand, would require sums which are both conditioned and mereologically varying, and yet unique. And finally, Simons's view makes reference to sums which are conditioned, non-unique, and mereologically varying.³²

³² See Chisholm 1976, Ch.3 (89ff.); Appendix A (138ff.); van Inwagen 1987; 1991 (first sections to section 9) Simons 1987, Section 5.2 (177ff.); Ch. 9 (324ff.).

4.3.2 *Conditioned Sums and Non-unique Sums*

By proposing either conditioned sums or non-unique sums, a neoclassical mereologist offers a way to resolve some of the most pressing problems which face the classical mereologist. The complaint put forward in connection with the Aggregate Argument, that a classical sum survives the dispersal of its parts, while typical wholes do not, does not apply to a conditioned sum - if it is conditioned with respect to relations that are disrupted by the dispersal of the parts. If a tree, for example, is a conditioned sum of its cells, in that its existence depends on the cells being appropriately juxtaposed to one another, then it is no wonder that the tree ceases to exist on the dispersal of its cells, for dispersal of the cells entails that the appropriate relations of juxtaposition cease to hold between the cells, so that the conditioned sum ceases to exist.

Similarly, a neoclassical mereologist may accept the claim that a gold ring and the lump of gold which constitutes it are distinct from one another, while insisting that either of them is a sum of the relevant gold molecules. For by allowing non-unique sums, such a theorist may take both the lump of gold and the ring to be sums of the same molecules, without admitting that these sums are identical to one another.

The means whereby a neoclassical mereologist introduces conditioned sums and non-unique sums are rather straightforward. As regards relationally conditioned sums, one simply rejects the classical principle of Universal Existence of Sums (P7), and introduces in its place qualified principles of existence of sums. That is, one forgoes the assumption that whenever there exist entities, the *xs*, there exists a sum of those entities. Instead, one assumes that whenever there exist entities, the *xs*, such that the *xs* are conditioned in a particular way (e.g. bear certain specified relations to one another), there exists a sum of the *xs*.

As regards non-unique sums, one begins by rejecting the classical principle of uniqueness of sums (P6), as well as the principle according to which *is an improper part of* is identified with *is identical to* (P1) (for reasons explained above). Instead, the neoclassical mereologist may rely, again, on the notion of a conditioned sum, to lay down conditions under which different sums exist of the same parts, where the

existence of each of these sums is dependent, respectively, on the preservation of different conditions.

For example, assuming that the *xs* are an appropriately large number of gold molecules, it may be laid down that a sum of these molecules exists, if the molecules are so juxtaposed that they occupy a continuous region of space. This sum may be identified as what we refer to by using the term 'lump'. Furthermore, it may be laid down that a sum of the molecules exists, if the molecules are arranged so as to occupy a ring-shaped region of space, and this sum may be identified as what we refer to by using the term 'ring'. Clearly, under some conditions, both the former and the latter relations hold between the *xs*. Since, however, under some conditions the former relations hold and the latter do not, the former sum will be claimed to be distinct from the latter sum. Since P6 and P1 are rejected, claiming this is not inconsistent with the neoclassical assumptions.

4.3.3 *Mereologically Varying Sums*

The complaint against classical mereology, that it cannot provide an account of mereologically varying wholes, can be met in neoclassical theory by introducing mereologically varying sums.

Suppose a certain tree has survived a forest fire. Some of the cells which belonged to it at *t* (before the fire) have been destroyed in the fire, however. These assumptions are not consistent with taking the tree to be a classical mereological sum. If the sum of the tree's cells at *t* is conceived along classical lines, this sum cannot survive the destruction of any of its parts. The assumptions are consistent, however, with taking the tree to be a mereologically varying sum, one which may be said to have different parts at different times.³³

The introduction of mereologically varying sums, within a neoclassical framework, is not as straightforward as the introduction of conditioned sums or of

³³ Mereologically varying sums *need* not have different parts at different times. Rather, they are entities which belong to a *kind* of sums, some of which have different parts at different times, while others do not.

non-unique sums. Peter Simons elaborates the notion, or more precisely *several* notions, of mereologically *varying* sums in Chapter 5 of *Parts*. The first step involves replacing the temporally unmodified mereological predicate 'is a part of' with the temporally modified 'is a part of at t ', as a primitive of the formal system.³⁴ Other temporally modified predicates, such as 'is a proper part of at t ', 'is disjoint from at t ', and 'overlaps at t ', are defined in terms of 'is a part of at t '.³⁵ In terms of these predicates, notions of mereologically varying sums can be explicated.

My presentation of the notion of a mereologically varying sum closely follows that of Simons.³⁶ The main difference is in the emphasis I place on the relation *is a sum-at- t* , which I explain in the next paragraph. Although this relation is only mentioned in passing by Simons, it will be seen that it is central to the various notions of merologically varying sums which he discusses. Emphasis on the role of this relation allows it to be clearly seen that the complex array of types of mereologically varying sums which Simons presents are closely related to one another, being different versions of one basic type of mereologically varying sums.

I take the notion of a mereologically varying sum to be the notion of an entity which (for some t) bears the relation *is a sum-at- t* to some entities.³⁷ This relation can be defined as follows (note the use of the temporally modified predicate 'overlaps at t ')

D3 y is a sum-at- t of the x s =_{def.}

y exists at t , and for all z , z overlaps y at t iff z overlaps one of the x s at t

Let us suppose, for example, that a certain church is built out of bricks, and that bricks are the only entities out of which it is built (that is, we are ignoring mortar, beams, etc.). Suppose we treat the church as a mereologically varying sum which exists at t , and suppose that bricks a_1, \dots, a_n are all the bricks which are parts of the church at t . Then the church is a sum-at- t of a_1, \dots, a_n , for the church exists at t , and something overlaps the church at t iff it overlaps one of a_1, \dots, a_n at t .

³⁴ See Simons 1987, 177-9.

³⁵ See Simons 1987, 179.

³⁶ See Simons 1987, 183-6.

³⁷ The notion of a sum-at- t presented here corresponds to that of Simons's ' Su_t ' sums. See Simons 1987, 184.

The notion of a mereologically varying sum allows for such a sum to comprise different individuals at different times. That is, if $t \neq t'$, z may be a sum-at- t of the x s, and a sum-at- t' of the y s, where the x s are not (severally) identical to the y s. For example, it may be that the church, of the above example, is a sum-at- t' of bricks b_1, \dots, b_n , where t' is a time later than t , and where at least one of these bricks (say, b_i) is not one of a_1, \dots, a_n . This would be the case if, for example, fifty years after the church was built repairs were carried out, and several of the original bricks were replaced by new ones.

To specify a mereologically varying sum y , one needs to specify, for any time t , which x s (if any) are such that y is a sum-at- t of those x s. Simons suggests two basic ways in which this might be done:³⁸

(1) The first is by means of a list of names. Thus one might speak of a mereologically varying sum of a and b , where a and b are individuals which exist over periods of time which may or may not overlap one another (' a ', ' b ' are the names in the list relevant to this case). Simons considers this way of specifying a sum only with regard to cases involving *two* summands, that is, only with regard to cases where a *binary* sum is involved.³⁹ He indicates that there are two different types of entity which might be intended by such specification.

He calls one type that of 'SU' sums. An 'SU' sum of a and b is an entity which exists whenever *either* a or b exists:

D4 c is an 'SU' sum of a and b =_{def.}
 for all t and for all x ,
 x overlaps c at t iff
 x overlaps either a or b at t

The other type he calls that of 'SM' sums. An 'SM' sum of a and b is an entity which exists whenever *both* a and b exist:

³⁸ See Simons 1987, 183-6.

³⁹ See Simons 1987, 183-4.

D5 c is an 'SM' sum of a and $b =_{\text{def.}}$

for all t and for all x ,

x overlaps c at t iff

both a and b exist at t and x overlaps either a or b at t

If d_1, \dots, d_n are entities, then an 'SM' sum of those entities is an entity which exists only at such times (if at all) in which every one of the ds exists. Clearly, therefore, the notion of an 'SM' sum is not the notion of a mereologically varying sum, for an 'SM' sum may not have different parts at different times (ignoring any mereological variation taking place in any of the ds themselves). Thus, although 'SM' sums are explicated in temporally modified terms, they are not mereologically varying sums.

By contrast, an 'SU' sum may well be mereologically varying. In particular, if c is an 'SU' sum of d_1, \dots, d_n , such that c exists both at t and at t' (where $t \neq t'$), and if some of the ds which exist at t do not exist at t' , then c will have parts at t which it does not have at t' .

(2) The second way in which a mereologically varying sum might be specified is by means of descriptive phrases, rather than lists of names. Thus one might speak, for example, of a mereologically varying sum of the schoolteachers, 'schoolteacher' being the relevant descriptive phrase.

Simons shows that the account of sums specified in this way is more complex, because there is a greater scope for ambiguity regarding the type of entity that might plausibly be intended. Simons mentions three such types, although it is clear that more might be distinguished.⁴⁰

One type is that of an entity such that, at any time t in which it exists, it is a sum-at- t of all persons who live at t and who are schoolteachers *at one time or another*. More precisely and generally, taking 'is an F at t ' to be a temporally modified descriptive predicate (such as 'is a schoolteacher at t '), this type of sum may be defined as follows:

⁴⁰ See Simons 1987, 184-6.

D6 a is an 'SU' sum of $Fs =_{\text{def.}}$

for all x and for all t

x overlaps a at t iff

for some y and for some t' , y is an F at t' and x overlaps y at t

A second type is that of an entity such that, at any time t in which it exists, it is a sum-at- t of all persons who live at t and who are schoolteachers at some specified time t' (t' might be, for example, the time of an utterance involving the phrase 'sum of the schoolteachers'). This type of sum may be defined as follows:

D7 a is an 'SU' sum of Fs at $t' =_{\text{def.}}$

For all x and for all t

x overlaps a at t iff

for some y , y is an F at t' , and x overlaps y at t

A third type of entity which might be intended by means of 'sum of schoolteachers' is that of an entity such that, at any time t in which it exists, it is a sum-at- t of all persons who live at t and who are schoolteachers at that time, t . In order to avoid ambiguity, it is important to distinguish here between two ways in which 'schoolteacher' might be predicated.

If we say that for all t there exists x such that x is a schoolteacher, we might either mean (i) that at any time t there exists someone who is a schoolteacher *at that time*, or (ii) that at any time t there exists someone who is a schoolteacher *at some time or another*. To distinguish these two types of predication, let us abbreviate 'is a schoolteacher at that time' by 'is an actual schoolteacher', and 'is a schoolteacher at some time or another' simply by 'is a schoolteacher'. Let us call a predicate involved in the former type of predication an *actual* predicate.

Taking 'is an actual F ' to be a temporally modified actual descriptive predicate, the third type of sum (of those specified by means of descriptions) may be defined as follows:

D8 a is an 'SUM' sum of actual $Fs =_{\text{def.}}$

for all x and for all t

x overlaps a at t iff

for some y , y is an actual F and x overlaps y at t .

Inspection reveals that any of these three notions of a sum (i.e. any of the three presented above which involve specification by means of descriptive phrases rather than by means of lists of names) are notions of mereologically varying sums. A sum of schoolteachers, understood as an entity of either of these three types, may clearly be an entity y such that, if $t \neq t'$, y has parts at t which it does not have at t' .

This concludes my look at a variety of types of mereologically varying sums, types which are distinguished as being specified either by means of a lists of names or by means of descriptive phrases (interpreted in one of three ways).

Chapter 5

Traditional Higher-grade Wholes as Sums

A recurrent and striking feature of reflection on wholes and parts, both in ancient and modern times, is the attempt to draw a distinction between what might be described as lower-grade and higher-grade types of whole. Perhaps the earliest example of such a distinction is the one which is at the source of the present work, namely, Plato's distinction in *Theaetetus* between a complex which is different from all its parts and a complex which is identical to all its parts.⁴¹ A more familiar ancient example is found in Aristotle's distinction between substances and heaps.⁴² In early modern times, Leibniz distinguishes similarly between monads and monadic aggregates.⁴³ In the 19th and early 20th centuries, two notions in particular developed into central representatives of this perennial attempt to explicate a higher-grade type of whole: the notion of an organic whole, and the notion of a Gestalt.⁴⁴ The latter two are fairly described as the main traditional modern notions of higher-grade wholes.⁴⁵

⁴¹ See Plato *Theaetetus* 201c ff.

⁴² See for example Aristotle *Metaphysics* 1041b11-31.

⁴³ For a discussion of Leibniz's distinction, see Rescher 1967, chs.7 and 9.

⁴⁴ For references regarding these notions, see discussion below.

⁴⁵ Terms other than 'organic whole' and 'Gestalt' are often used in modern discussions to designate wholes which may be described as of a higher-*grade*. Husserl, for example, uses the term 'pregnant whole' (see Husserl 1901, Investigation III, sec. 22); Russell uses the term 'unity' (see Russell 1903, 139); among thinkers associated with the Gestalt school we sometimes find 'dynamic whole' and 'functional whole' (see discussion in Nagel 1952); more recently Simons uses 'integral whole' (see Simons 1987, Ch. 9) and Lowe uses 'integrate' (see Lowe 1989, Ch. 6).

With regard to all of these notions it can be argued, as I do with respect to organic wholes and Gestalts, that having their characteristic features is compatible with being a sum, neoclassical if not classical. In fact, there are complex and often close connections between

The predicate 'is of higher-grade' is used loosely in connection with such notions of wholes, in the following sense: such notions present the relations between the properties of a whole and the properties of (and relations between) its parts as a more complicated, less straightforward affair than do the contrasting lower-grade notions. Motivation for developing higher-grade notions, to contrast with lower-grade ones, can be found in many and varied contexts.

To take one example, minor changes in the relations between the parts of a heap of car-parts are likely to be less consequential for the properties of the heap than comparable changes in the relations between the parts of a car are likely to be for the properties of the car. If one assumes that the difference between the sensitivity of the (properties of the) car and that of the (properties of the) heap to changes in the relations between their respective parts is better viewed not as being a matter of degree, but rather as being a matter of fundamental difference of kind, it would seem plausible to attempt to develop a corresponding theoretical distinction between two different notions of a whole, a higher-grade and a lower-grade one.

What I would like to suggest in the current section is that the neoclassical mereological framework is flexible enough to admit sums which have any of the features commonly associated with higher-grade notions of wholes. That is to say, to claim that a whole is a sum (i.e. either a classical sum or a neoclassical one) is compatible with claiming that it is a higher-grade whole. I wish to discuss this claim in any detail only with respect to two of the types of higher-grade whole mentioned above, namely, organic wholes and Gestalts.

I am not attempting to establish conclusively that all higher-grade types of whole, nor even organic wholes or Gestalts in particular, can be adequately accounted for as types of either classical or neoclassical sum. To do so would require a full-length study of its own. My more limited task is merely to indicate that such an account of higher-grade wholes is plausible. This serves two purposes. First, it is meant to show how traditional higher-grade notions fit in from the perspective of the mereologies - classical and neoclassical - that we have been considering, and thus to render our theoretical picture less fragmented.

the notions of an organic whole and of a Gestalt and those associated with the other terms just mentioned. It will not be possible, however, in the present framework, to trace these connections in detail.

Secondly, it is meant to disincline us from assuming that familiar higher-grade notions are likely to offer adequate responses to problems raised (in the next section) in criticism of the conception of wholes as sums. If, for example, the principal features by which an organic whole is characterised are fully compatible with treating such a whole as a sum, it should not be expected that the notion of an organic whole could offer an adequate response to criticisms directed at the foundation of the conception of wholes as sums.

Section 5.1

Organic Wholes and Gestalts

5.1.1 *Organic Wholes*

The notion of an organic whole gained a wide currency around the turn of the last century, both in philosophical circles and outside them.⁴⁶ As examples of prominent philosophers during this period for whom the notion played a central role, we may mention Nietzsche, Bradley, McTaggart and Whitehead. The immediate sources for this notion are usually traced to Goethe, Kant and Hegel, although it seems clear that much older ideas were influential in its development, notably ideas due to Aristotle and Leibniz.⁴⁷ Indeed, from the 18th century onward the notion appears to have been an important expression of a current in the *Zeitgeist* associated with a reaction to dominant mechanistic views typical of the Enlightenment.⁴⁸

⁴⁶ Moore speaks disapprovingly of the widespread, and often very vague, use of the term 'organic whole' in Moore 1903, 27ff. See also Benziger 1951.

⁴⁷ Benziger (1951) traces Coleridge's discussions of organic wholes back to Leibniz, through a complex genealogy involving Schlegel, Karl-Phillip Moritz, Herder, Goethe, and Leibniz, and considers alternative claims tracing Coleridge's ideas back to Schelling and Kant. Moore takes discussions of organic wholes current in his time (e.g. in Bradley and McTaggart) to have their principal source in Kant and Hegel.

Shusterman (1988) takes Nietzsche's understanding of the notion of an organic whole to be a radical development from views put forward by Aristotle in *Poetics* 1451a30-35.

The view according to which Aristotle's substances should be understood as organic wholes can be found in Bogaard 1979, 12, and similarly in Ross 1924, 219.

⁴⁸ Max Oelschlaeger comments on the role of the cognate notion of an organism as follows:

Since the publication of Descartes's *Discourse on Method* (1637) and *Meditations* (1641) and Newton's *Principia Mathematica* (1687), the mechanistic model has dominated natural science. The Cartesian-Newtonian paradigm enjoys cognitive hegemony in the modern world, displacing any aesthetic, religious, or philosophical claim to insight or knowledge. ... Modernism draws, perhaps unconsciously but absolutely, a boundary between an objective or scientific and a poetic or aesthetic view of nature.

However,

Even though Modernism has intellectually ruled "reason" in the Western world for some four hundred years, challenges to its cognitive adequacy have been numerous. ... Collectively considered, the critics of Modernism engendered an

A central intuition underlying the notion of an organic whole concerns a certain "unity" (I place the term in quotes to distinguish it from 'unity' in the sense explained in Chapter 1, 1.1.3.3 and 1.2.5) which wholes of some but not all types allegedly possess, a "unity" understood somehow as the perceptible presence - equally in each of the many parts of the whole - of one and the same causally responsible item. The type of causation involved here is sometimes assumed to be efficient, sometimes teleological. Living organisms, and particularly plants, are usually taken as paradigmatic wholes of this type - hence the use of the term 'organic'. Benziger, in his discussion of the role of the notion in Coleridge's thought, explains as follows:

The very use of the word *organic* to describe a work of art is a metaphor. It is a comparison of the unity of a work of art to that unity which ordinary men imagine they perceive in a tree, a unity which is the expression of one indwelling force or spirit.⁴⁹

Goethe, for example, says of the Strassburg cathedral that

its great harmonious masses [are] filled with life even to the countless small details, like the works of eternal nature, all form, all contributing to the whole, down even to the most minute fiber.⁵⁰

The "unity" of organic wholes is further understood as rendering the parts of such wholes entities which are in some sense not independent, either with respect to one another or with respect to the whole of which they are parts.

Thus the parts of an organic whole are sometimes taken as having properties which are dependent on the properties of the other parts or of the whole, sometimes as depending for their own existence on the existence of the other parts, or of the whole, and sometimes as being incapable of being understood or even conceived of, independently of one another or of the whole. Furthermore, it is sometimes unclear whether the necessity characterising these dependencies is

opposition between two rival ideas of nature: *the idea of nature-as-a-machine* as against that of *nature-as-an-organism*.

See Oelschlaeger 1991, 97-8.

⁴⁹ Benziger 1951, 33.

⁵⁰ Benziger notes Vol. XXX, 9 of *Goethe's Sämtliche Werke*, Jubiläumsausgabe, Stuttgart, 1902-7.

meant to be causal, ontological or logical. An example in which claims regarding the presence of a single causally responsible item in the many parts are combined with claims regarding the non-independence of those parts can be found in Thomas Wartenberg's account of Hegel's conception of the world as an organism.⁵¹

As we shall see presently, various features (such as having parts which are functionally or ontologically dependent on one another, or having properties which are emergent with respect to the properties of the parts) have been described in the attempt to explain in more precise terms what the claim that a whole is organic amounts to. It is debatable whether a statement of any or all of those features does justice to the fundamental intuition that gives rise to the notion of an organic whole. My claim that the notion of an organic whole is adequately treated as falling within the framework of the conception of wholes as sums (classical or neoclassical), however, is only made with respect to organic wholes *in as far* as they are taken to be adequately characterised by some or all of those proposed features.

5.1.2 *Gestalts*

The introduction of the notion of a Gestalt is chiefly credited to Christian von Ehrenfels, who discussed it in his article 'Über Gestaltqualitäten'.⁵² The notion is particularly familiar in connection with the major influence it has exerted on modern psychology, especially through the work of Max Wertheimer, Wolfgang Köhler and Kurt Koffka. It has also been influential, however, in other fields, ranging from the social sciences to artificial intelligence.⁵³

By contrast with the case regarding the notion of an organic whole, that of a Gestalt arose not in the context of describing paradigmatic individuals, such as

⁵¹ See Wartenberg 1993, 102ff., and esp. 108.

⁵² See Ehrenfels 1890. Ehrenfels takes his views to be a development of some observations made by Mach. Simons claims that "the same concept was elaborated and exploited at the time by two other former students of Brentano, Meinong and Husserl, who used the terms 'founded content' and 'figural moment' respectively for what Ehrenfels called a 'Gestalt-quality' (Simons 1987, 356).

⁵³ For a brief survey of ways in which the notion of a Gestalt has featured in a variety of scientific fields, see Palmer and Rock 1990.

trees or cathedrals, or works of art, such as poems or paintings, but rather in the context of describing types of content of sensation or of perceptual experience. Ehrenfels's discussion begins with a consideration of two examples, that of the shape in which several points are arranged in space, and that of a melody, understood as a musical "shape" in which several notes are arranged in time.⁵⁴

According to the leading theory of perception at the time (most notably associated with Helmholtz), perceptual experience can be adequately analysed into primitive sensations which have qualities such as colour and brightness (in the visual case), or pitch (in the aural case) as their contents.

In the case of visual experience, for example, each point of the visual field is thought of as associated with a primitive sensation - the sensation of a certain colour and a certain brightness at that point. A typical visual experience is viewed as involving many such sensations taking place at the same time (just as the sensation of heat in one finger may be simultaneous with the sensation of coldness in another finger). Furthermore, it is assumed that such a visual experience can simply be *equated* with these many simultaneous primitive visual sensation^s, which together account for the whole field embraced by the experience.

Suppose, for example, many red dots are arranged in the shape of a circle. According to that theory, experiencing the quality associated with the perception of the circular shape is understood simply as the simultaneous experiencing of the primitive qualities associated respectively with the sensation^s of each of the points.

Similarly, experiencing the quality associated with the perception of a melody is understood simply as the consecutive experiencing of the primitive qualities associated respectively with the sensation of each of the notes.

To use Ehrenfels's terms, such an account takes a shape to be a "sum" (or a "mere sum")⁵⁵ of dots, and a melody to be a "sum" of notes, and in this respect it seemed to him not true to the familiar experiences of hearing melodies or of seeing shapes. He urged that shapes and melodies, and a great variety of other types of item, be recognized as qualities which, notwithstanding the clear sense in which

⁵⁴ A central meaning of the German term 'Gestalt' is 'shape'.

⁵⁵ The terms 'mere sum' and 'sum', as used by Ehrenfels, are placed in double-quotation marks, to distinguish this use from the one we have adopted, in connection with the treatment of classical and neoclassical mereology.

they depend on respective primitive qualities, are not to be analysed as "sums" of such primitive qualities in the way suggested. He used the term 'Gestalt-quality' to describe such qualities.

In later usage among Gestalt theorists, the term 'Gestalt' was generally preferred to 'Gestalt-quality'. This is partly to do with the fact that the term became commonly used to designate not qualities of this type, but rather *entities* which are possessed of qualities of this type.⁵⁶ In what follows, I shall use 'Gestalt' in the latter sense.

Taking Gestalts and "sums" as two types of whole;⁵⁷ the most noteworthy distinguishing feature of the former as contrasted with the latter, in Ehrenfels's view, concerns the way in which the similarity between two Gestalts is not determined, or at least not completely determined, by the similarities between their parts. Let us call this condition, which is met by Gestalts on Ehrenfels's account, 'Ehrenfels's criterion'. This criterion, he claims, is not answered by wholes which are described as "sums". For he takes the similarity between two "sums", by contrast, to be completely determined by the similarities between their parts.⁵⁸

For example, if a "sum" of notes has notes as its parts which are respectively perfectly similar to the notes which are parts of another "sum", then the two "sums" are perfectly similar. By contrast, if a Gestalt has notes as its parts (a melody being an example of such a Gestalt) which are respectively perfectly similar to the notes which are parts of another Gestalt, the two Gestalts may still be very dissimilar to one another. The similarity between melodies clearly depends not only on the identity of the notes which are their parts, respectively, but also on the order in which those notes are played. Ehrenfels assumes that the similarity between "sums" of notes does not depend on the order of the notes.

⁵⁶ The ambiguity as between quality and entity possessed of the quality is analogous to the ambiguity in the use of 'structure', either to designate (collectively) the relations between the parts of an entity, or to designate an entity, the parts of which are thus related (in the latter sense, a bridge, for example, can be described as a structure).

⁵⁷ For the purpose of discussing the views of Gestalt theorists, we may take 'whole' to have a wider application than we have agreed to adopt otherwise in the present work, so that not only concrete entities, but also shapes and melodies, might be described as wholes.

⁵⁸ More precisely, if S is a sum whose parts are $a_1 \dots a_n$, and S' is a sum whose parts are $b_1 \dots b_n$, then the similarity between S and S' is a function of (and only of) the similarities between a_1 and b_1 , ..., and between a_n and b_n .

We find, then, that there are two aspects to Ehrenfels's insight. One concerns the claim that the similarities between some wholes is not completely determined by the relevant similarities between their parts. Wholes of this type are called Gestalts. The other aspect concerns the claim that wholes of this type can be contrasted with wholes of another type, the similarities between which *are* completely determined by the relevant similarities between their parts. Wholes of the second type are called "sums" (or "mere sums").

It has been demonstrated by Kurt Grelling and Paul Oppenheim that the first aspect of Ehrenfels's account admits of a satisfactory precise formulation. In their article, 'The Concept of Gestalt in the Light of Modern Logic', they have shown how a feature grounding the similarities between two wholes can be expressed, a feature which is determined not only by the similarities between the relevant parts of the two wholes, but also by the relations between those parts.⁵⁹

It is more difficult to explicate convincingly the second aspect of Ehrenfels's account. The claim that some wholes comply with Ehrenfels's criterion seems to be uncontroversial. What is controversial, however, is the claim that some wholes do *not* comply with that criterion.

For the natural question to ask is, what might count as an example of a "mere sum"? Whenever one hears a sequence of notes, these notes are obviously heard in a specific temporal order, and the similarity between this sequence and another will be at least partly determined by the order of the notes in the sequence. And a similar remark would apply to any case in which a whole is seen, the parts of which are dots on a sheet of paper. Turning from the consideration of the contents of sensation and perception to the consideration of concrete entities, again no clear examples of wholes which would be judged to be "sums" on Ehrenfels's criterion seem to be forthcoming.

The main response of Gestalt theorists to this difficulty was effectively to abandon Ehrenfels's criterion, while attempting to preserve the intuition suggested by his seminal essay concerning the contrast between a higher-grade type of wholes and a lower-grade type, which the notion of a Gestalt is meant to explicate.⁶⁰ Gestalts are on this view higher-grade wholes, typical examples of which are found

⁵⁹ See Grelling and Oppenheim 1938. See also Simons 1987, 354-360.

⁶⁰ See, for example, the discussion in Köhler 1929, Ch. 4.

in the organic world (e.g. the brain of a human being), but not only in the organic world (e.g. an electrically charged capacitor). An account of the notion of a Gestalt, on this view, would require that the theorist identify particular types of characteristics of a whole, and especially particular types of relations between the parts of a whole, which mark the distinction between wholes that are Gestalts and wholes that are not Gestalts. The characterisations that were actually offered turned out to be, by and large, variations (though often interesting variations) on the familiar characterisations of organic wholes.

Among theorists not belonging to the Gestalt school a different response could be found. The notion of a Gestalt was interpreted not so much as applying to higher-grade wholes, as contrasted with lower-grade ones, but rather as applying to any whole whose identity depended on the relations between its parts. On this view, a sand dune, or a heap of old car-parts, are no less Gestalts than is a car, or for that matter an organism. Indeed, Grossmann, for example, claims that any concrete whole is a Gestalt in this sense.⁶¹ He thinks that the only reasonable interpretation for Ehrenfels's "sums" is one which takes them to be classes, that is, abstract entities.⁶²

Simons also does not consider the notion of a Gestalt to mark a distinction between a whole which would usually be considered to be higher-grade, such as an organism, and a whole which would usually be considered to be lower-grade, such as a sand dune.⁶³ According to his view, both an organism and a sand dune are Gestalts, though he leaves open the possibility that some wholes exist whose identity does not depend on the relations between their parts, and thus that wholes exist which are not Gestalts. The notion of a classical mereological sum, rather than what I have described as a (neoclassical) conditioned sum, would apply to such non-Gestalts, and would provide an adequate characterisation of them.⁶⁴

⁶¹ See Grossmann 1983, 242.

⁶² See Grossmann 1983, 241. On the plausibility of interpreting Ehrenfels's "sums" as classes, see also Nagel 1952, 143.

⁶³ Simons does however take it that the notion of a Gestalt, as developed, for example, by Köhler, is valuable in helping to direct us to the important problem, "what *makes* something a natural (or other) complex, which entails giving a schematic account of the constitutive interrelations of parts characteristic of such complexes." See Simons 1987, 324-6.

⁶⁴ Note however that in Simons's usage, by contrast with the one I have adopted (according to which 'sum' applies in general to wholes in classical and neoclassical mereology) 'sum' does not apply to what I have called conditioned sums.

In what follows, the discussion is confined to the notion of a Gestalt as put forward by the Gestalt theorists. I shall therefore focus on presenting features which have been proposed by Gestalt theorists, features by means of which the notion of a Gestalt is marked out as a notion of a higher-grade whole. Regarding the claim that I argue for below, namely that the notion of a Gestalt is adequately treated as falling within the framework of the conception of wholes and sums, a comment is in place which is parallel to the comment made above with regard to organic wholes: I make this claim only with respect to Gestalts *in so far* as they are taken to be adequately characterised by some or all of those proposed features.

To conclude, it can be seen that notwithstanding their distinct intuitive roots, the notions of an organic whole and of a Gestalt came to be associated interchangeably with one or more features taken from a certain pool of features of higher-grade wholes. I shall now turn to state briefly the main features belonging to this pool, and argue that *sums* may be proposed which have any of these features, thus showing that conceiving of a whole either as an organic whole or as a Gestalt is compatible with conceiving that whole as a sum.

Section 5.2

Features Associated with Organic Wholes and Gestalts

In *Principia Ethica*, G. E. Moore offers a critique of the notion of an organic whole, by gathering what he takes to be the main claims that have been variously made by theorists who utilize the notion, claims that indicate the main features which have been associated with the notion of an organic whole in his view.⁶⁵ Thus he says that the description of an entity as an organic whole "is generally understood to imply" one of the following three claims:

- (1) The entity has parts which "are related to one another and to [the whole] itself as means to end." ⁶⁶
- (2) The entity is such that its parts "have a property described in some such phrase as that they have 'no meaning or significance apart from the whole'." ⁶⁷
- (3) The entity is such that its "value is different from the sum of the values of [its] parts." ⁶⁸

If we compare Moore's review with those offered by some other commentators on organic wholes and on Gestalts, we find that if these three features are appropriately generalized, they do, by and large, represent the main accounts of the two notions.⁶⁹

⁶⁵ See Moore 1903, 27-36.

⁶⁶ Moore 1903, 31.

⁶⁷ Moore 1903, 31.

⁶⁸ Moore 1903, 31; see also 29.

⁶⁹ See Nagel 1952; Grossmann 1983, Ch. 5; Hartshorn 1942; Wartenberg 1993, Sec. II; Shusterman ¹⁹⁸⁸, ; Köhler 1929, esp. Ch. 4. ;

5.2.1 *Wholes Involving Types of Causal Dependence Between the Parts*

As regards (1), Moore takes the claim to be intended as a characterization of a feature which living organisms are commonly assumed to possess, whereby the continued existence of some part is a necessary condition for the continued existence of some or all of the other parts. Thus he says that a whole that is organic in this sense is one in which some of the parts exhibit "a relation of mutual causal dependence on one another".⁷⁰

Moore has in mind a relation such that the *existence* of a part causally depends on the existence and proper functioning of another part. More generally, we may speak of relations such that either the existence of a part *x*, or its having certain properties, or its bearing certain relations (to some of the other parts of the whole), causally depends on the existence of some other part *y*, or *y*'s bearing certain properties or certain relations (to some of the other parts of the whole).

If the properties of, or relations between, the parts of a whole exhibit such causal dependencies, they are said to be *dynamically* determined. Wholes whose parts have properties and stand in relations which are dynamically determined are sometimes described as *functional* wholes.⁷¹

Moore notes that the relation of means to end might be interpreted otherwise than as such efficient causal dependence. It might, for instance, be interpreted as involving some teleological relation between the behaviour of one part and other parts or the whole.

Indeed, Charles Hartshorn's notion of an organic whole can be stated as precisely such an alternative interpretation of (1). The essential feature of an organic whole according to him is not that the parts of such a whole are causally dependent on one another, but rather that they "serve as 'organs' or instruments to a purpose or end-value inherent in the whole".⁷² On Hartshorn's account, the criterion for a whole's being organic is that the behaviour of (all) the parts of the whole is

⁷⁰ Moore 1903, 32.

⁷¹ See Köhler 1929, Ch. 4, esp. 111-120; See also Nagel 1952, 147, making reference (notes 6 and 7) to Wertheimer, Koffka and Lewin.

An interesting formal account of the notion of a functional whole is proposed by Grelling, in Grelling 1987, and discussed in Simons 1987, 342ff.

⁷² Hartshorn 1942, 127.

teleologically related to the existence and behaviour of the whole, and not that it is causally related (in the sense of efficient causation) to the existence and behaviour of the whole or to any of the other parts.

5.2.2 *Wholes Involving Types of Ontological Dependence between the Parts*

Moore's second sense of 'organic whole', defined by (2), is further elaborated by him as follows:

It is supposed that just as the whole would not be what it is but for the existence of the parts, so the parts would not be what they are but for the existence of the whole.⁷³

He adds that this is distinguished from the feature of causal dependence between parts, expressed in (1) above:

... this is understood to mean not merely that any particular part could not exist unless the others existed too ... , but actually that the part is no distinct object of thought - that the whole, of which it is a part, is in its turn a part of it.⁷⁴

What Moore seems to be saying is that in the case of (2) the existence of the parts is not conceivable except in conjunction with the existence of the other parts (and, presumably, in conjunction with appropriate relations to those other parts). The existence of a part does not merely depend *causally* on the existence of some or all of the other parts, or on their being related in some appropriate way to one another. A stronger kind of necessity links the existence of a part and the existence of the other parts. Shusterman paraphrases Moore's understanding of (2) as follows:

The reciprocal dependence of the parts is not simply causal but logical. The very essence, features, or character of any part is somehow constituted or

⁷³ Moore 1903, 33.

⁷⁴ Moore 1903, 33.

modified by its interrelations with other parts in the whole, so that *it would be inconceivable and would make no sense to speak of any part as existing as that same part on its own*. For there is no essence or nature of the part to speak of apart from its interrelations in the whole.⁷⁵

Leaving aside questions as to whether Shusterman adequately captures Moore's intentions here, it may be noted that the relations between the parts of an organic whole according to this view belong to types of *ontological dependence*.

In general outline, by saying that entity *x* is ontologically dependent on entity *y* we mean that necessarily, if *x* exists then *y* exists. According to this definition, relations of causal dependence of the type described by Moore in connection with notion (1) of an organic whole also count as relations of ontological dependence. However, the claim that *x* is ontologically dependent on *y* is usually taken to imply that the necessity involved is not merely nomological but rather metaphysical or logical.⁷⁶

In chapter 8 of *Parts*, Peter Simons offers an extensive treatment of ontological dependence, demonstrating that the notion admits of a variety of important distinctions. His treatment is based, as he emphasizes, on Husserl's theory of *foundation* and of *pregnant wholes*.⁷⁷ Simons distinguishes between a *weak* notion of ontological dependence, according to which dependence of an entity on one of its parts counts as ontological dependence, and a *strong* notion, according to which only dependence on an entity which is not a proper part may count as a relation of ontological dependence.⁷⁸ Furthermore, he distinguishes between *rigid* and *generic* dependence. If it is necessary for the existence of an entity *a* that some entity or

⁷⁵ Shusterman 1988, 382 (italics mine).

⁷⁶ See, in this connection, Simons' remarks (in Simons 1987, 295) on the fuzzy boundary between causal and ontological dependence.

⁷⁷ See Simons 1987, 290-323. Husserl's theory is found chiefly in the *Logical Investigations*, Investigation III. For treatments of Husserl's theory that are alternative to Simons', see Fine 1995.

⁷⁸ I am ignoring here a subtlety in Simons' discussion. He defines strong dependence on the basis of a definition of "absolute independence": *a* is absolutely independent if it is possible that: *a* exists and that for all *x*, if *x* exists then it is *included* in *a*, where 'included' might be substituted by 'is a part of' (either as governed by the classical assumption P1 or alternatively, in the way Simons suggests on p.112) or by 'is weakly included in' (a notion that arises in Lesniewskian logic, which I do not treat in the present work). Strong dependence can thus be defined somewhat differently, according to the substitution chosen.

another of sort *F* exists, then *a* is said to depend *generically* on entities of the sort *F*. If, on the other hand, the existence of *a* necessitates the existence of a *particular* entity *b*, then *a* is said to depend *rigidly* on *b*.

I shall not engage in the present work with the question whether the explication of ontological dependence which Simons offers adequately captures the intentions of Moore, or those of Nietzsche (on Shusterman's account). In Shusterman's case, at least, the view that the parts of a whole are ontologically dependent on one another is associated with a radical reconception of the nature of those parts. A part, it seems to be urged, is to be conceived as *constituted* by its relations with other parts. It is not an entity of the sort I have been talking about, which stands in relations to other entities of this sort; rather, it simply is those relations. And the whole also is not conceived of in the familiar way I have been assuming, but rather as a sort of network of relationships.

I am doubtful whether such a metaphysical view can be made sufficiently intelligible, and in any case I will not attempt to elucidate it here. My claims regarding sums whose parts are ontologically dependent on one another are confined to a metaphysical framework which takes the category concrete, non-attributable entities (i.e. either individuals, or pluralities; see Section 1.1) as basic and not reducible to the category of relations.

5.2.3 *Wholes Involving Types of Emergent Properties*

Moore's third sense of 'organic whole', associated with claim (3), can be put by saying that a whole is organic if its intrinsic value is an emergent property with respect to the properties of its parts, on one or another understanding of the notion of an emergent property. To interpret (3) more precisely, several notions of an emergent property must be distinguished from one another. In characterising these notions, I build on the familiar distinction between determinables and determinates.⁷⁹

⁷⁹ The distinction between determinables and determinates is due to W. E. Johnson (see Johnson 1921, 174). See also Rosenberg 1995; Simons 1987, 343f.

Any property which belongs to a particular range of contrary properties is called a '*determinate*'. For example, *is red* is a determinate, belonging to the same range of contrary properties as does the determinate *is green*. A range of such contrary properties is often found to be associated with a certain corresponding property which is called a '*determinable*', a property possessed by anything which possesses any of the determinates belonging to that range. For example, *is coloured* is a determinable corresponding to the range of determinates which includes *is red* and *is green*. Relations admit of a similar distinction between determinable and determinates. Thus *is 1'' away from* and *is 2'' away from* are determinates to which the determinable *is at a distance from* corresponds. At present, however, I shall focus on determinable and determinate *properties*. If F is a determinable, we may speak of the determinates associated with it as '*F-determinates*'.

5.2.3.1 Quantitatively Emergent Properties

One way to understand the claim that *has intrinsic value* (as possessed by the whole) is an emergent property is as follows.

First of all, one assumes that *has intrinsic value* is a quantitative determinable. A determinable F is *quantitative* if the F -determinates are distinguished from one another as being of a certain *degree*, a degree which can be fixed numerically, with greater or less precision. Examples of quantitative determinables are: *has weight*, *has volume*, *has electrical charge*. With respect to such determinates a precise notion of the *arithmetical sum* of these properties may be defined: If F_n and F_m are F -determinates, such that F_n is of degree n and F_m is of degree m , then the arithmetical sum of F_n and F_m is F_{n+m} , an F -determinate of degree $n+m$.

Now, suppose that in a certain case both the whole and each of the parts have a quantitative determinable F . Suppose that the whole has F -determinate F_n , and that the arithmetical sum of the F -determinates of the parts is F_k , where k is different from n . In such a case we may say that F_n is *quantitatively emergent* with respect to the F -determinates of the parts. Let us assume that the same point is

expressed by saying that the *F*-ness of the whole is quantitatively emergent with respect to the properties of the parts.

Taken in one way, therefore, Moore's claim (3) can be explicated as the claim that the intrinsic value of the whole is quantitatively emergent with respect to the properties of the parts. This is one sense in which it might be said that *has intrinsic value*, as possessed by the whole, is an emergent property.

5.2.3.2 *Fundamentally Emergent Properties*

A second way of understanding the claim that *has intrinsic value*, as possessed by the whole, is as follows.

Suppose a whole has a determinable *F* which *none* of its parts have. In such a case we may say that *F* is *fundamentally emergent* with respect to the properties of the parts. This might be expressed alternatively by saying that the *F*-ness of the whole is fundamentally emergent with respect to the properties of the parts.

Authors who made extensive use of the notion of emergence earlier in this century seem primarily to have had fundamental emergence in mind.⁸⁰ Jaegwon Kim summarises the "doctrine of emergence" associated with those authors as follows:

The doctrine of emergence, in brief, is the claim that when basic physicochemical processes achieve a certain level of complexity of an appropriate kind, genuinely novel characteristics, such as mentality, appear as "emergent" qualities.⁸¹

Mentality is not seen as a determinable, determinates of which are possessed both by a complex whole and by each of its parts, but rather as a determinable which is possessed by the whole and yet by none of its parts. This is what Samuel Alexander means when he speaks of mental processes as having a "distinctive quality of mind

⁸⁰ The view that wholes may have emergent properties is particularly associated with Samuel Alexander and with C. Lloyd Morgan, in whose views the notion played a central role. Kim mentions in addition G. H. Lewes and C. D. Broad as representatives of emergentist thought. See Kim 1993, 134.

⁸¹ Kim 1993, 134.

or consciousness", as contrasted with corresponding neural processes.⁸² It is also implicit in C. Lloyd Morgan's claims regarding "levels" which are discernible in an organism, each level associated with a "distinguishing quality". He conceives of an organism as a "pyramid with ascending levels", and says:

Near its base is a swarm of atoms with relational structure and the quality we may call atomicity. Above this level, atoms combine to form new units, the distinguishing quality of which is molecularity; higher up, on one line of advance, are, let us say, crystals wherein atoms and molecules are grouped in new relations of which the expression is crystalline form; on another line of advance are organisms with a different kind of natural relations which give the quality of vitality ...⁸³

Indeed, the emergentism of Alexander and of Lloyd Morgan involves a stronger claim than that organisms, mineral crystals, and other such entities have properties which are fundamentally emergent with respect to the properties of their parts. Calling the properties with respect to which an emergent property is emergent the '*base properties*', one finds that emergentism is further associated with the following assumptions:⁸⁴ (1) that both the entities which have emergent properties and any of their parts are *physical*;⁸⁵ (2) that an emergent property is not *reducible* to its base properties; and (3) that an emergent property *supervenes* on its base properties.⁸⁶

The emergentists, therefore, have conceived of emergent properties as a particular species of what I am calling fundamentally emergent properties.⁸⁷ The claims that I make below regarding the compatibility of the conception of wholes as sums with the view that a whole has emergent properties is meant to apply to

⁸² See Alexander 1927, Vol. 2, 5-6 (quoted in Kim 1993, 345).

⁸³ Lloyd Morgan 1923, 35 (quoted in Kim 1993, 345).

⁸⁴ See Kim 1993, 344ff.

⁸⁵ I do not go into the question what this assumption precisely amounts to. For the emergentists, at any rate, one of its principle expressions was the rejection of Cartesian minds, as well as of Driesch's notion of an entelechy and Bergson's *élan vital*. A (very) brief discussion of the latter two notions can be found in Feldman 1995.

⁸⁶ Varied aspects of reduction and supervenience are discussed in Kim 1993 (see Chs. 4 and 8 in particular).

⁸⁷ It seems clear that the claim that a property is *fundamentally* emergent is consistent with the denial of any, or all, of the assumptions (1), (2) and (3) in its respect, though I shall leave this point unargued here.

fundamentally emergent properties in general (as well as to emergent properties of the other types that I distinguish), and therefore in particular to emergent properties as conceived by the emergentists.

5.2.3.3 *Other Notions of Emergence*

In addition to the notions of a quantitatively emergent property and a fundamentally emergent property, further notions of an emergent property may be distinguished.

Let us say that if a determinable is not quantitative, in the sense explained above, it is qualitative. Examples of qualitative determinables are *has colour*, *has shape*. If *F* is a qualitative determinable possessed both by the whole and by the parts, then the *F*-ness of the whole is neither fundamentally emergent, nor quantitatively emergent with respect to the properties of the parts. However, various relations between the *F*-determinate of the whole and the *F*-determinates of the parts may be considered as constituting other types of emergence. To take the most obvious possibility, it might be suggested that if the whole has an *F*-determinate which none of the parts do, then the *F*-ness of the whole is *qualitatively emergent* with respect to the properties of the parts.

I shall not explore further in this direction, however. For one thing, it seems not to lead to additional plausible interpretations of Moore's claim (3). For another, it does not seem that cases of this type should raise new difficulties not already raised by quantitative and fundamental emergence, as regards the question whether wholes which have such emergent properties can be conceived of as sums.

In conclusion of this discussion of emergence, I would like to make note of a very weak notion of an emergent property, which I call a 'trivially emergent property': the property of a whole is trivially emergent with respect to the properties of the parts if, simply, it is different from any of the properties of the parts. Clearly, both fundamentally emergent properties and quantitatively emergent properties are trivially emergent. Indeed, sometimes the claim that a

property is emergent is understood as the claim that it is trivially emergent in this sense.⁸⁸

However, it is easy to see that all concrete wholes possess trivially emergent properties (any concrete entity has a weight, for example, that none of its parts do), and so the possession of trivially emergent properties cannot serve as a criterion for distinguishing between higher-grade and lower-grade wholes (i.e. between organic and inorganic wholes, or between Gestalts and "sums", etc.).

To summarise the discussion of the present Section to this point - three general features have been noted, by means of which organic wholes and Gestalts have been traditionally characterised:

- F1 The whole has parts which are causally dependent on one another (where different types of causal dependence might be intended).
- F2 The whole has parts which are ontologically dependent on one another (where this ontological dependence might be characterised in different ways).
- F3 The whole has properties which are emergent (either quantitatively or fundamentally) with respect to the properties of the parts.

5.2.4 *The Slogan 'the whole is greater than the sum of its parts'*

A feature which has often been proposed as characterising such wholes is commonly expressed by the slogan "the whole is greater than the sum of its parts".

Perhaps the most natural interpretation of this slogan is to take it to express a version of the view which has already been noted above, namely that Gestalts, or organic wholes, have properties which are quantitatively emergent with respect to

⁸⁸ Grossmann suggests an understanding of the claim in this way, when he says that "A structure ... is also more than the class of its parts in that it may have properties which none of its parts has. There are, as it is sometimes put, emergent properties." (Grossmann 1983, 246), or when he says that a certain "spatial figure has a certain property, its shape, which none of its part has", in virtue of which he takes the shape to be an emergent property (See Grossmann 1983, 249).

the properties of their parts.⁸⁹ In connection with the claim that an *F*-determinate of a whole is quantitatively emergent with respect to the *F*-determinates of the parts, we speak of an arithmetical sum of the *F*-determinates of the parts, and give a clear arithmetical sense to the claim that the *F*-determinate of the whole is different from this arithmetical sum of *F*-determinates of parts. One type of case in which this claim would be true is that in which the *F*-determinate of the whole is *greater* than the sum of *F*-determinates of the parts, where again 'greater' has a clear arithmetical sense, namely, 'numerically greater'.

It is perhaps also natural to take the slogan more widely as expressing the view that wholes of the relevant types have properties which are either quantitatively or fundamentally emergent with respect to the properties of their parts. There is a certain analogy between these two notions of emergence, and this analogy may ground a somewhat metaphorical use of the terms 'greater' and 'sum', according to which a whole that has a property which is fundamentally emergent with respect to the properties of the parts can be said, somewhat loosely, to be "greater than the sum of the parts".

Grossmann offers an alternative interpretation of the slogan. As we have noted (in 5.1.2) he takes 'sum' as used by Ehrenfels's as designating a class. A concrete whole is clearly different from the class of its parts, and this, Grossmann thinks, is what is meant by saying that the whole is greater than the sum of its parts.⁹⁰ However, the assumption that a whole is greater than the sum of its part, in this sense, does not offer an additional criterion for counting a whole as being higher-grade. Indeed, this feature follows directly from concreteness, and so is possessed both by lower-grade and by higher-grade wholes.

Mention of this slogan, however, gives us the opportunity to clarify a possible misunderstanding. It might be thought that organic wholes, or Gestalts, were held (by authors who have taken them to embody a notion of a higher-grade whole) to

⁸⁹ Indeed, the slogan interpreted in this sense is suggested by Moore's formulation of claim (3).

⁹⁰ Grossmann's use of 'sum' seems not to be consistent, though. In the first paragraph of Sec. 101 (Grossmann 1983, 242) he adds that the whole is "more ... than the sum of its nonrelational and relational parts", and provides an argument for this claim which would be both superfluous and misleading had he used 'sum' here in the sense of 'class'.

be greater than, or at least different from, the sum of their parts, where 'sum' is understood in the sense used in classical or neoclassical mereology.

In the first place, however, it is very unlikely that Gestalt or organic whole theorists who made the claim expressed in the slogan had the technical sense of 'sum' (associated with mereological theories) in mind.⁹¹

In the second place, merely to make such a claim is not enough, of course. When we seek for grounds offered for justifying the claim, we are led to variants of precisely those features that we have already noted, F1, F2 and F3. In the next Section, however, I argue that for a whole to possess any or all of these features is fully compatible with conceiving it as a sum (either classical or neoclassical). Thus, there seem to be no good reasons for accepting the claim that a whole is different from (or greater than) the (classical or neoclassical) mereological sum of its parts.

⁹¹ I have already noted above the difficulties involved in determining what Ehrenfels means by contrasting Gestalts with "mere sums"; in particular, both classical and neoclassical sums do not fit the conditions required by Ehrenfels of his "sums".

Note also that the theories of organic wholes and Gestalts which we have been looking at were all fully developed by the 30s of this century, while the notion of a classical sum (not to speak of the notion of a neoclassical sum) was not familiar in the West before Leonard and Goodman's article of 1940 (Leonard and Goodman 1940). On this latter point, see Küng 1967, 107.

Section 5.3

Organic Wholes and Gestalts as Sums

Having argued that the features by which either organic wholes or Gestalts have been held to be characterised can be summarised as F1, F2 and F3 of the previous subsection, I now turn to showing that these features are all compatible with the conception of wholes as either classical or neoclassical mereological sums.

I do this by assuming with regard to each of these features, that wholes of a certain type *W* (where for '*W*' either 'organic whole' or 'Gestalt' may be substituted) are characterised as having this feature. It may then be asked whether a whole's being of this type is or is not compatible with its being either a classical sum or a neoclassical sum. Once this is done, the way to generalize our conclusions to positions according to which *Ws* have two of these features, or all three of them, should be clear, and will not require separate treatment. For convenience, we may speak of a whole of type *W* simply as a *W*.

5.3.1 *Sums with feature F1*

Consider first the claim that a *W* is characterised by feature F1. Thus it is claimed that a whole belongs to type *W* if it has parts which are causally dependent on one another. Now this claim is susceptible to two distinct interpretations. It might be interpreted either as the claim

I1 a whole is a *W* if and only if it has this feature at some time *t*

or as the claim

I1* a whole is a *W* if and only if it has this feature *essentially*

According to the first interpretation, if a whole has feature F1 at some time *t*, then it is a *W*, even if it does not have the feature at some other time *t'* (and even if it might not have the feature at *t* or *t'*). According to the second interpretation, a whole is a

W only if it has, and moreover must have, this feature at all times in which exist. Clearly, if a whole is a W on interpretation I1*, then it is also a W on interpretation I1.

I am not aware of an explicit statement, either on the part of authors expounding the notion of an organic whole or on the part of those expounding the notion of a Gestalt, which would indicate whether they have I1 or I1* in mind. I1* would seem to be the more natural one. On either interpretation, however, the possession of this feature by a whole is compatible with that whole's being a sum. For on interpretation I1, both a classical sum and a neoclassical sum may be Ws. While on interpretation I1*, although a classical sum may not be a W, a neoclassical sum may be one. We can see this as follows.

According to interpretation I1, a classical mereological sum may be a W. For suppose many entities exist which are causally related to each other at time *t*. Classical mereological assumptions guarantee that a sum of those entities exists. This sum is an entity which has feature F1 at *t*, and therefore according to I1 it is a W. If at some other time, *t'*, the entities disperse and cease to be causally related to one another, then their sum still exists. At *t'*, therefore, the sum will not have feature F1. This, however, does not conflict with the sum's being an organic whole according to I1. According to I1, then, the possession of feature F1 by a whole is compatible with that whole's being a classical mereological sum. And since the neoclassical assumptions are weaker than the classical ones, the possession of F1 by a whole according to I1 is *a fortiori* compatible with its being a neoclassical sum.

According to interpretation I1*, a classical sum cannot be a W, if we assume that the parts of this sum might disperse and cease to be causally related to one another, and yet continue to exist. However, a *neoclassical* sum may be a W according to this interpretation. For suppose again that many entities exist which are causally related to one another at time *t*. It is consistent with neoclassical assumptions (though not with classical assumptions) that these entities have a sum only if they are thus related to each other. Such a sum is described as a conditioned sum. It seems correct to say that a conditioned sum is essentially conditioned as it is. Therefore, the sum is a W according to I1*. The possession of feature F1 by a whole, according to I1*, is thus compatible with that whole's being a neoclassical mereological sum.

5.3.2 *Sums with feature F2*

Let us now consider the claim that a *W* is characterised by feature *F2*. In this case it is claimed that a whole belongs to type *W* if it has parts which are ontologically dependent on one another. Again the claim admits of two interpretations, either as

I2 a whole is a *W* if and only if it has this feature at some time *t*

or as

I2* a whole is a *W* if and only if it has this feature essentially

As before, I will not attempt to determine which of these interpretations better represents the views of the relevant organic whole theorists or Gestalt theorists. It is sufficient for us to see that on either interpretation the possession of *F2* by a whole is compatible with that whole's being a sum. Indeed, on either interpretation the possession of *F2* by a whole is compatible both with its being a classical sum and with its being a neoclassical sum. We can see this as follows.

Suppose the *xs* are entities which are all ontologically dependent on one another. On classical assumptions, there exists *y* which is a sum of the *xs*. Moreover, on classical assumptions, any sum has the parts it has essentially. Thus *y* is such as to have, essentially, parts which are ontologically dependent on one another. Therefore, on interpretation I2*, *y* possesses feature *F2*, and yet *y* is a classical sum. On interpretation I2*, then, the possession of *F2* by a whole is compatible with its being a classical sum. And this implies compatibility (regarding classical sums) on interpretation I2 as well.

The conclusion of the last paragraph implies also that the possession of *F2* by a whole is compatible with its being a neoclassical sum, on either interpretation, because of the weakness of the neoclassical assumptions relative to the classical ones.

5.3.3 *Sums with feature F3*

Finally, let us turn to the claim that a *W* is characterised by feature *F3*. According to this claim, a whole belongs to type *W* if it has properties which are

emergent (either quantitatively or qualitatively) with respect to the properties of the parts. The claim might be interpreted either as

I3 a whole is a *W* if and only if it has this feature at some time *t*

or as

I3* a whole is a *W* if and only if it has this feature essentially

Again, leaving aside the question, which of the interpretations is historically more faithful to the views of theorists who propounded the notion of an organic whole or that of a Gestalt, we find that on either interpretation, the possession of F3 by a whole is compatible with the whole's being a sum. In particular, we find that on interpretation I3, both classical and neoclassical sums may be *Ws*, and that on interpretation I3*, while a classical sum cannot be a *W*, a neoclassical sum may be one.

Classical mereology, in assuming that for any *xs* there exists a *y* which is the sum of the *xs*, places no explicit constraints on the properties which *y* may have, given the properties of the *xs*. Implicitly, of course, there are clear constraints with regard to one type of properties which *y* has, properties which may be designated as 'mereological'. For example, if one of the *xs* is green, then the mereological assumptions imply that *y* has the property *has a green part*. It seems that apart from such mereological properties, however, no constraints applying to the properties of a sum vis-à-vis the properties of its parts can be derived from the mereological assumptions.

For example, if *y* is a sum of three parts, each of which weighs one kilogram, there is nothing in the classical assumptions to rule out that *y* weighs fifty tons. And similarly, if *y* is a sum of so and so many trillions of molecules, none of which possess the property of consciousness, the classical assumptions are silent about whether *y* does or does not possess this property. If the sum does possess the property of consciousness at time *t*, then it possesses feature F3, and therefore is a *W*, according to interpretation I3. And since being a *W*, on I3, is compatible with being a classical sum, it is also compatible with being a neoclassical sum.

However, it is highly plausible to assume that if the sum of those molecules has the property of consciousness at time *t*, then its having this property depends

(for example) on the molecules being spatially related to one another in some particular way.⁹² If at some other time t' each of the molecules is thousands of kilometres away from any of the other molecules, then it would seem very unlikely (to say the least) that y could have the property of consciousness at t' . And yet according to classical assumptions, as long as the molecules exist, whether arranged as the parts of a living organism or not, their sum y exists.

On plausible (non-mereological) assumptions, therefore, it seems that although a classical sum may, on interpretation I3, have feature F3, it cannot on interpretation I3*.

Neoclassical assumptions, however, allow us to propose, for example, that a sum of the molecules exists only if those molecules are arranged in a particular way (this would be a conditioned sum). It would be an essential feature of such a sum that its parts are molecules which are arranged in that particular way. If this condition regarding the molecules is sufficient for the sum to have the property of consciousness, then such a sum would have the property of consciousness essentially, and would thus have feature F3, and would be a W , according to interpretation I3*.

That a neoclassical sum might be a W according to interpretation I3* may be demonstrated more directly however. The condition associated with a conditioned sum may pertain not only to relations between the parts, but also to properties of the parts. In particular, since according to neoclassical assumptions (as well as classical assumptions) a whole is a part of itself, the notion of a conditioned sum includes that of a sum which possesses a certain property essentially. In particular, it may be assumed that a sum possesses an emergent property essentially. For example, it is consistent with neoclassical assumptions to assume that a sum exists which essentially possesses the property *is conscious*.

This shows that a whole's being a W , on interpretation I3*, is compatible with its being a neoclassical sum.

⁹² This assumption is weaker than the assumption that the emergent property supervenes on the properties of and relations between the parts. To say that it supervenes on those properties and relations is to say that the sum has this property *if and only if* the parts have those properties and are thus related to one another. While I am merely assuming on that the sum has this property *only if* the parts have those properties and are thus related to one another.

We may conclude that according to a variety of familiar accounts of the notions of an organic whole and of a Gestalt, the claim that a whole is an organic whole, and the claim that the whole is a Gestalt, are compatible either with the claim that the whole is classical sum, or at least with the claim that it is a neoclassical sum. Our discussion suggests, therefore, that conception of wholes as sum is flexible enough to accommodate traditional modern conceptions of higher-grade wholes, and is fully compatible with them.

Chapter 6

Criticism of the Notion of a Neoclassical Sum

Having looked at the advantages which neoclassical mereology offers as contrasted with its classical ancestor, in allowing for conditioned sums, non-unique sums, and mereologically varying sums, and in providing what might seem to be an adequate framework for accounting for traditional higher-grade notions of wholes, I now turn to consider some weaknesses of this flexible theory.

It may be noted, in the first place, that a classical theorist will be unhappy with neoclassical mereology, if she takes it to be one of the main purposes of mereology to provide an alternative to the theory of classes for the purposes of analysing statements of common discourse. For example, Goodman proposes to analyse

Every species of dog is exhibited.

as

For every x , if x is a species of dog then some y is a dog and is part of x and is exhibited.

where he says that species of dog "may be regarded as certain discontinuous whole composed of dogs",⁹³ instead of as a class whose members are dogs of that species. Without the principle of Universal Existence of Sums (P7), Goodman would be unjustified in assuming the species of dog, understood in this sense, exists.⁹⁴ With Lesniewski, as with Goodman, mereology originated as an attempt to do away with

⁹³ See Goodman 1951, 29.

⁹⁴ For further discussion of the project of analyzing common discourse by mereological means, see Goodman 1951, 26-33.

ontological commitment to classes, so that adherence to the classical principles is essential to their project. It is not, however, essential to the different project of accounting for the pre-theoretical notions of whole and part, and someone who acknowledged the existence of classes would see little point in constraining mereology so as to serve the logical roles that Lesniewski and Goodman require.⁹⁵

Granted that ours is the latter project, reflection on the conception of wholes as sums, which neoclassical mereology proposes, leads us nevertheless to discover a multitude of inadequacies of this conception, and suggests that the elaboration of an alternative conception of wholes is desirable, at least in connection ^{with} \vee some of the important contexts in which the notion of a whole features.

In what follows I discuss some of the problematic aspects of the three main types of neoclassical sum. Although the problems I point to do seem to me to be serious, and although I am doubtful whether they could be adequately resolved within the framework of the conception of wholes as sums, I do not take the discussion to constitute a *refutation* of the neoclassical position. To attempt to offer such a refutation would require a much lengthier discussion, and would in any case be beside my purpose.

The main interest in discussing these problems, from my point of view, is to provide an instructive backdrop against which to evaluate the alternative conception of wholes as Unities which I present in the next Part. Thus I have attempted to focus on particular aspects of the conception of wholes as sums which give rise to (what I take to be) substantial difficulties, aspects with respect to which (as we shall see) the conception of wholes as Unities differs. Because it differs in this way from the conception of wholes as sums, the difficulties to which I point here either do not arise at all in the context of a theory of Unities, or if they do arise, they have straightforward solutions.

⁹⁵ See on this point Simons 1987, 101-105.

Section 6.1

Conditioned Sums

In classical mereology it is assumed that for any xs there is a y such that y is a sum of the xs (P7). Denying this assumption in neoclassical mereology allows one to introduce the notion of a conditioned sum. This is done by assuming that for any xs , there is a y such that y is a sum of the xs if and only if the xs fulfil a certain condition (e.g., a certain relation obtains between the xs).

At first sight, this neoclassical proposal is very attractive. It seems that wholes are often structured entities. At least in the case of what I have called *prima facie* individuals, the parts of the whole are related to one another in such a way that collectively they exhibit a certain structure. The existence of the whole in such cases clearly depends on the preservation of that structure, in the following sense: the whole cannot be made up of those parts unless the relations between them comply with certain constraints. The introduction of conditioned sums seems to be a promising way to express this observation about wholes, for the existence of such sums depends precisely on the preservation of some structure which is exhibited by their parts.

A closer look at the way structured wholes are accounted for as sums, however, reveals a peculiar problem. One seems bound to render the conditions under which parts have a sum as strikingly, and implausibly, sensitive. The neoclassical theorist is oddly without freedom to impose conditions as she deems appropriate in this respect.

Let us imagine a human organism h who is made up, at t , of a great many molecules $m_1 \dots m_n$ (I shall take 'the ms ' to abbreviate ' $m_1 \dots m_n$ '). It would seem that these molecules, considered collectively, are capable of making up a whole of a certain type, when they are appropriately related to one another. If the relation *make up* is interpreted as the relation *compose*, as it is in both classical and neoclassical mereology, then we should say that the ms are capable of composing h when they are appropriately related to one another. And to say that they compose h is equivalent to saying that h is their sum.

Now suppose an additional molecule (not identical to m_1 or ... or m_n) , m^+ , is juxtaposed to the ms , without otherwise changing the conditions fulfilled by the latter molecules, and suppose this juxtaposition results in m^+ being an additional part of h . Noting that the xs compose u if and only if u is a sum of the xs , and recalling the definition of a sum, whereby the claim that u is the sum of the xs is equivalent to the claim that for all z , z overlaps u iff z overlaps one of the xs , we directly conclude that after the addition of the extra molecule the original molecules can no longer compose the organism. For if we assume that the ms and m^+ (together) compose h , then it follows that the ms do *not* compose h . Again, if we assume that to make up is the same as to compose, this means that the original molecules, $m_1 \dots m_n$, no longer make up h .

This seems to be quite a remarkable predicament. Countless trillions of molecules, taken collectively, possess a noteworthy capacity to make up a human organism, if appropriately conditioned, and yet this capacity can be disrupted by the juxtaposition of the most insignificant, virtually undetectable particle. The molecules, it turns out, instantaneously cease to make the organism up, the moment the additional molecule has become a part of the organism.

When m^+ becomes a part of h , the multitude of molecules the ms would seem to undergo one of the following two dramatic changes, even though we assume that neither their properties, nor the relations between them, change in any way. Possibly, (1) those molecules cease to make up an entity altogether. This would be consistent with neoclassical assumptions, since those do not include the principle of Universal Existence of Sums (see 4.1.2, principle P7).⁹⁶ Alternatively, (2) those molecules change from making up a human organism to making up an entity of a radically different kind. This entity, which would clearly be a proper part of h , could hardly be an organism of any kind, let alone a *human* organism (interesting exceptions aside, proper parts of organisms are generally not themselves organisms).⁹⁷

⁹⁶ That this is what happens in such a case is van Inwagen's view (1981).

⁹⁷ See in this connection Simons's criticism of Chisholm's assumption that a proper part of a table may be a table (Simons 1987, 192-3). Simons notes that his point had already been made by Wiggins and by Quine.

Note also that although I assumed that the radical change in the collective ability of the original molecules was engendered by a newly added molecule, it might just as well have been an electron, or a quark. And similarly note that an analogous argument could be formulated taking a far larger entity than a human organism - say, a whale, or perhaps even a planet or a galaxy (if these are accepted as *bona fide* entities).

The assumption that an entity is a conditioned sum seems to imply, therefore, that the collective capacity of a great number of particles to make up an entity would be radically disrupted in the presence of an additional particle. The existence of a sum of the original particles depends not only (plausibly) on relations between those particles, but also (implausibly) on whether or not an additional single particle is present in the vicinity. To put it otherwise - before m^+ was added, the ms possessed the collective attribute *are such as to make up a human organism*.⁹⁸ The approach of m^+ caused the other molecules to cease to have this attribute. On the face of it, it seems surprising that one molecule should have such an effect on so many molecules.

It might be responded that the impression that a single molecule exerts a disproportionate causal influence, on the assumption that the organism is a conditioned sum, is the result of a misleading description of the circumstances we were considering. Granted, the approach of the new molecule results in the original molecules ceasing to make up the organism. However, they still have an attribute after the addition of the new molecule, which is very much like the attribute they had before. Before they had the attribute *are such as to make up a human organism*. After they had the attribute *are such as to contribute overwhelmingly to making up a human organism*. These attributes, it is suggested according to this response, are very similar - indeed, virtually indiscernible, precisely in accordance with the limited influence of a single new molecule.

I don't think that this response is satisfactory, however. If the ms (conditioned appropriately) were such as *not* to make up a human organism, but the ms (identically conditioned) together with m^+ *would* make up a human organism, then it would seem appropriate to say that the ms "contribute" to making up a human

⁹⁸ Such attributes are described as multi-grade relations. See Section 1.3 above.

organism. Indeed, in such a case the addition of m^+ does not change the attributes of the ms in any radical way. For the ms were neither making up h before the addition of m^+ , nor were they doing so after. Both before and after they could be described as contributing towards making up h . Before, this was without result, because an additional required contribution, however small, was lacking. After, the contribution was effective, in co-operation with the contribution from molecule m^+ .

The case we have been examining, however, is different. In our case, the ms (conditioned appropriately) *do* make up a human organism, just as much as but the ms (identically conditioned) together with m^+ would do. The following analogy may illuminate the relationship between these two types of cases.

Let us assume that if a single bullet pierces the heart of a person - no matter where in the heart - that person dies. Let us also assume that anyone who shoots a person with the result that a bullet pierces the heart of the person, can be said to have caused the person's death. Now, if two gunmen shoot the person so that two bullets pierce the person's heart simultaneously, it would not be appropriate to say of one of the gunmen that he merely *contributed* to causing the person's death. Both, it would seem, are fully responsible causally (we might perhaps say, "overlappingly" responsible).

Developing the example, let us assume that if a hundred bullets or more simultaneously pierce the heart of a giant, the giant dies, and those who shot the hundred bullets (where each gunman is assumed to fire no more than one bullet) have collectively caused the giant's death. If less than a hundred bullets do so, the giant does not die. If one hundred and one bullets are shot, and simultaneously pierce the heart of the giant, then it would be (in parallel to the case of the last paragraph) inappropriate to say of a hundred gunmen that they (collectively) *contributed* to the giant's death. Any hundred gunmen (among the hundred and one) can be said to be (collectively) fully responsible for the giant's death. Any ninety nine or less (among the hundred and one), by contrast, *can* be said to have merely contributed to the giant's death.

Our case of the molecules and the human organism, after the addition of a molecule, is somewhat similar to the case of the giant and the hundred and one bullets. Just as any hundred gunmen - taken collectively - are *fully* responsible causally, and do not merely contribute, to the giant's death, so the molecules, the

ms, seem to be fully responsible, in respect of making up a human organism, and do not merely contribute to making up such an organism.

I conclude that the response to the problem I raised is not satisfactory. It fails to pay attention to the distinction between a case in which many molecules (taken collectively) merely contribute to making up an organism, and a case in which many molecules (taken collectively) are fully responsible, in respect of making up the organism. The neoclassical conception of a conditioned sum allows that the original molecules have such full responsibility in respect of making up the organism only before the new molecule was added, and not after. In this way it compels us - if we acknowledge the difference between merely contributing to making up, on the one hand, and fully making up, on the other hand - to accept that the addition of a molecule has a totally disproportionate influence on the attributes of the original molecules taken collectively. I take this to be an indicative weakness of the proposal to conceive of organisms (for example) as conditioned sums.

Section 6.2

Non-unique Sums

It is among the advantages of neoclassical mereology that it allows for the same entities to have more than one sum. As I have noted in Section 4.3, the rejection of principles P6 and P1 has the result that it is consistent with the remaining principles that for some y , z , and xs , y is a sum of the xs and z is a sum of the xs and yet y is not identical to z . That is to say, it is neoclassically consistent to assume that two distinct entities are superposed with one another, and in particular that two distinct entities are at the same place at the same time.⁹⁹

The flexibility of the neoclassical framework in this respect is an advantage particularly because there are compelling reasons to think that distinct wholes may be superposed with one another at various stages of their career.¹⁰⁰ If (as it would seem difficult to deny) a bronze statue does not exist after the lump of bronze it is made of has been reshaped into a bronze ball, and if (again, as it would be difficult to deny) the lump of bronze has existed both before and after this reshaping, then on Leibniz's principle of indiscernibility of identicals it would follow that the statue

⁹⁹ I have noted above that 'superposition' is defined so that the claim that u and v are superposed with one another is equivalent to the claim that for some xs , u is a sum of the xs and v is a sum of the xs .

By saying that u and v are at the same place at the same time, I mean that they are contained in precisely the same region of space at the same time, without implying that they share parts at all (and thus without implying that they are superposed with one another). Arguably, a mind might be in the same place at the same time as a body, without their sharing any parts, and in particular without their being superposed with one another.

¹⁰⁰ The most influential modern proponent of the view that distinct wholes may be superposed with one another has been David Wiggins (See especially Wiggins 1968 and Wiggins 1980). For some evidence regarding the wide acceptance of the view in recent years, see Burke 1992, 12(n.1), evidence which leads Burke to dub this view 'the standard account'. The view is not new, however. John Locke, for example, accepted that wholes that are not of the same kind (e.g. a "mass of matter" and a horse which it constitutes at some time t) may be superposed with one another (he speaks of being in one and the same place and time, rather than of the sharing of parts; though the latter seems to have been implied). See Locke 1689, Ch. XXVII (esp. 330-1).

and the lump are not identical to one another. For after the reshaping it is the case that the lump is spherical, but it is not the case that the statue is spherical.¹⁰¹

In order to resist this inference, one would need to resort to one of several possible extreme measures, such as that of denying that statues, or lumps (or both) exist,¹⁰² or of denying that entities might be identical across time,¹⁰³ or of restricting, in one way or another, the scope of Leibniz's principle.¹⁰⁴ It must be admitted, however, that there is (at least initially) a strong intuitive inclination to resist this inference.

I will argue below that while the assumption that distinct entities may be superposed is not in itself objectionable, the assumption that distinct *sums* may be superposed is objectionable, for if two sums are superposed, they are indiscernible in a certain respect which renders the claim that they are distinct incoherent. Thus while there are good reasons to think that there are non-unique wholes (i.e. distinct wholes which are made up, at one and the same time, of exactly the same parts), there are also good reasons to think that there cannot be non-unique sums. My conclusion from this is that if the arguments of Wiggins and others, according to which the possibility of superposition must be accepted, are to be maintained, an alternative must be found to the conception of wholes as sums.

Before presenting that argument, however, I wish to consider an argument of David Lewis's which might be taken to constitute an objection to the claim there can be non-unique wholes, conceived as sums or otherwise. It will be useful to see why

¹⁰¹ An elegant version (illustrated by different examples) of the argument briefly presented in this paragraph can be found in Wiggins 1968.

¹⁰² On the view that Doepke calls 'The One-Many View', the lump (in this example) does not exist, strictly speaking. 'Lump' would be interpreted as (simultaneously) designating the many bronze particles. On the view he calls 'The Reductivist View', neither the lump nor the statue exist, strictly speaking. See Doepke 1982.

¹⁰³ This would still leave the objector to superposition, such as Lewis or Noonan, with an unresolved parallel problem that can be posed with regard to modal properties, which involve properties that the statue and the lump *might* have had at the time in question (rather than properties that they actually do have at times different from the time in question). For answers the objectors might offer to this problem, as well as criticisms of these answers, see e.g. Johnston 1992, Noonan 1993 (and references therein).

¹⁰⁴ Doepke 1982, 47-50, argues that avoiding the consequence of superposition by asserting that identity is relative to sort or to time amounts to an arbitrary restriction of Leibniz's principle. The view that identity is relative to sort is developed by Peter Geach (1980), and by Nicholas Griffin (1977). For criticisms of this view, see especially Wiggins 1980, Ch.1, and Lowe 1989, Ch.4., as well as Doepke 1982.

his stronger conclusion is not convincing, before arguing for the weaker conclusion that I propose.

6.2.1 *Lewis's Objection to "Unmereological" Composition*

David Lewis, in his article 'Against Structural Universals',¹⁰⁵ makes a point which, if acceptable, would constitute an objection to the suggestion that there might be non-unique sums. He comments as follows:

... how can two different things be composed of exactly the same parts? I know how two things can be made of parts that are qualitatively just the same - that is no problem - but this time, the two things are supposed to be made not of duplicate parts, but of numerically identical parts. That, I submit, is unintelligible.¹⁰⁶

Lewis's discussion suggests that the claim

- (1) y and z (where $y \neq z$) can both be composed of exactly the same x s.

can be understood in two ways, but that on either reading it is unacceptable. On one reading the term 'compose' (and cognates) is understood literally; on the other, it is understood metaphorically. Now, it seems to me that there is nothing in what Lewis says in criticism of the literal reading of (1) which indicates that this claim is *unintelligible*. Of the points he makes, the only ones that could be taken as an explanation why he takes (1) to be unintelligible are points that he makes in connection with the metaphorical reading of (1). However that may be, let us see how Lewis argues for the unacceptability of (1) on either reading, apart from the bald statement quoted above regarding its unintelligibility.

When he speaks of a *metaphorical* understanding of 'the x s compose y ', Lewis has in mind a possible extension of the application of such statements to cases in which the relation between any of the x s and y is *similar* to the relation *is a part of*. If I were to describe the relation between my parents and me, for example, as a kind

¹⁰⁵ Lewis 1986.

¹⁰⁶ Lewis 1986, 36.

of composition, I would be using 'compose' in such a metaphorically extended way. The relation *is a parent of* is similar in some formal respects, but not in others, to the relation *is a (proper) part of* (for example, both relations are transitive).¹⁰⁷

Lewis's basic argument is that if one asserts (1) using the metaphorical sense of 'compose' (denying (1) if the literal sense is used), one is asserting something that may well be true, but something that is in conflict with the further claim that the *xs* are *parts* of either *y* or *z*. For unless the *xs* compose *y* in the literal sense, there is no justification for claiming that the existence of *y* necessarily entails the existence of the *xs*. But, Lewis holds, the claim that the *xs* are *parts* of *y* implies that there *is* such a necessary connection between the *xs* and *y*. It follows that to use 'compose' metaphorically and yet to insist on a necessary connection between the *xs* and *y* is to admit a profound modal mystery into one's view.¹⁰⁸

If, on the other hand, one asserts (1) using the literal sense of 'compose', then one is asserting something that is simply false, because it is inconsistent with what Lewis takes to be a unique, uncontroversial account of the (literal) notion of composition. This account is of course the one offered by classical mereology, the only theory to which Lewis seems to think the term 'mereology' appropriately applies.¹⁰⁹ Indeed, he assumes that if composition, in the literal sense, were to be considered as consistent with (1), then it should be described as "unmereological" composition.¹¹⁰ He denies, however, that a coherent notion of such "unmereological" composition is available. If it were available, then (classical) mereological composition would be a special case of some more general notion. But he denies that there is a more general notion than the classical mereological one:

¹⁰⁷ See Lewis 1986, 38.

¹⁰⁸ Lewis describes the conception of structural universals as composed of the other universals (the instantiation of which is necessarily entailed by the instantiation of the former), in the case that 'compose' is used in a metaphorical sense, as 'the magical conception', referring to the inexplicability of the asserted necessary connection. See Lewis 1986, 41f.

¹⁰⁹ Indeed, in *Parts of Classes* he offers a detailed defence of the principles of classical mereology. See Lewis 1991, 75-81. Comments on Lewis's attitude to mereology can be found in Simons 1991.

For criticism of the supposed privileged status of classical mereology, see Doepke's comments in Doepke 1991, 393, as well as Simons 1987 (in general).

¹¹⁰ David Armstrong, whose notion of a structural universal is criticised by Lewis in the paper we are discussing, also uses of the term 'mereological' in the sense of 'classical mereological'. See Armstrong 1991, 190.

What is the *general* notion of composition, of which the [classical] mereological form is supposed to be only a special case? I would have thought that [classical] mereology already describes composition in full generality.¹¹¹

Now, as regards Lewis's point about the metaphorical use of 'compose', it has to be admitted that conceding such a use of 'compose' does render the alleged necessary connection between the existence of a whole and the existence of its ("unmereological") parts more mysterious than on a literal use of the term.

At least it is clear that we do not think that the existence of a person (at *t*) necessarily entails the existence of something *y* (at *t*) which is a parent of *x*, in the way that we do think that it entails the existence of something *z* (at *t*) which is a part of *x*. And in case of an entity *x* which is essentially unsusceptible to change, such as a universal, it does seem that *y*'s being a part (in a literal sense) of *x*, where *y* is also essentially unsusceptible to change, makes it quite clear why the existence of *x* (or its instantiation at *t*) necessarily entails the existence of *y* (or its instantiation at *t*). And this necessary connection would indeed be less clear if *y* was not a part (in the literal sense) of *x*.

Suppose then that we accept this point. We may still have doubts as to whether the concepts of whole and part do essentially involve such necessary connections (after all, it is clear that at least pre-theoretically, we use the terms 'whole' and 'part' in such a way that allows for the whole to survive the destruction of some of its parts). And even if we were convinced that these concepts do involve such necessary connections, we may still have doubts as to whether the modal features of the concepts of whole and part are as unmysterious as Lewis takes them to be. However, let us set these doubts aside, for the sake of argument. There is still no reason to think that this point constitutes an objection to neoclassical mereology, and in particular to its allowing for non-unique sums.

For there is no reason for neoclassical mereologists to concede that they are using 'compose' metaphorically, rather than literally. Lewis assumes that there is a unique, uncontroversial account of the notion of composition (taking 'compose' literally), the account offered by classical mereology. This, however, is precisely what the neoclassical theorist denies. The neoclassical theorist not only claims that

¹¹¹ Lewis 196, 39. Additions in square brackets mine.

there can be an alternative account, but goes ahead to offer one, in the shape of the neoclassical principles and various assumptions which are formally consistent with them regarding the existence of conditioned sums, non-unique sums and mereologically varying sums.

To the claim that they are using 'compose' metaphorically, neoclassical theorists might respond by pointing out that their account excludes such relations as *is a parent of* from counting as species of the relation *is a part of*, no less than the classical account does.¹¹² It seems fair to say, therefore, that the contrast between classical and neoclassical accounts of composition is not a contrast between accounts of literal and metaphorical senses of 'compose', but rather a contrast between different accounts of the literal sense of the term.

Lewis gives us no reason for investing classical mereology with the status of an unquestionable authority on the notion of composition (in the literal sense). As regards internal consistency, the principles of neoclassical mereology fare as well as those of classical mereology (in fact, since the former is the weaker system, its inconsistency would imply the inconsistency of the latter). And Lewis says nothing which would lead us to think that the assumption that non-unique sums exist is in any way in conflict with those principles. In particular, he does not explain what it is about the assumption that the same *xs* can simultaneously compose (in the literal sense) different entities which renders it unintelligible.

We conclude that Lewis's discussion does not cast a compelling doubt on the intelligibility of the notion of a non-unique sum, and thus does not provide good reasons for rejecting it.

6.2.2 *The Conflict between Non-identity and Present Indiscernibility*

There are three aspects to the state of the allegedly superposed statue and lump that one might find objectionable. One is that the two entities are supposed to occupy precisely the same region of space at the same time. A second aspect is that

¹¹² Note, for example, that Principle 5, of those that I listed, does not apply to the relation *is a parent of*.

there are some entities, the *x*s, of which both the statue and the lump are supposed to be sums. And a third aspect is that the two entities are supposed to be *presently indiscernible* at the time in question.

Two entities *x* and *y* are said to be presently indiscernible at some time *t* if they share all their *present* properties at *t* (if either *x* or *y* has a property at *t* which the other does not have, then they are said to be presently discernible at *t*). By describing a property of *x* or of *y* as present at *t* I mean that it is both *modally blinkered* and *temporally blinkered*. A property *F* is temporally blinkered if its instantiation at *t* does not entail the instantiation of any property at any time other than *t*. A property is modally blinkered if its instantiation in the actual world does not entail the instantiation of any property in any world other than the actual one. Properties that are either not modally blinkered or not temporally blinkered are *absent* properties.¹¹³

Simons takes it for granted that superposed pairs such as the statue and the lump are presently indiscernible.¹¹⁴ It seems to me, however, that the conclusion that they are presently indiscernible - assuming that they are made up of precisely the same entities - is inescapable only if the statue and the lump are conceived of (as by Simons) as sums. To see this, let us consider on what plausible grounds it might be argued that two superposed wholes are *not* presently indiscernible, suspending briefly the assumption that those wholes are to be conceived of according to neoclassical mereological principles.

Suppose *x* and *y* are superposed with one another, both of them occupying one and the same region of space *R*. It seems to me that the only compelling way to establish that some of the present properties of *x* are not among the present properties of *y* (i.e. that *x* is presently discernible from *y*) would be to indicate

¹¹³ Simons speaks of coinciding entities as *temporarily indiscernible* (Simons 1987, 213), by which he means that they share all their time-blinkered properties (at the time of coincidence), in the sense explained here. Both Doepke and Simons criticise Grice and Myro's (for reference see in Doepke 1982) proposal according to which entities which share their temporally blinkered properties at *t* are identical at *t*, though they might not be identical at some other time *t'* (thus Grice and Myro propose to relativise identity to time). It is clear, however, that in the context Grice and Myro, and Doepke, and Simons, all assume implicitly that coinciding entities share only temporally blinkered properties which are also modally blinkered. In order to make this assumption explicit, I use the expression 'presently indiscernible' (instead of 'temporarily indiscernible').

¹¹⁴ See Simons 1987, 211; 213.

among the present properties instantiated in R a pair of *incompatible* present properties. If P_1 and P_2 are incompatible present properties, both instantiated in R , then P_1 and P_2 cannot both belong to x (or to y). Unless such a pair could be found, there would seem to be no reason to deny that all of the present properties instantiated in R belong both to x and to y .

If we had a developed theory of wholes as (concrete) distributive classes, rather than as sums, a plausible candidate for such a pair of incompatible properties might readily be found. We recall that the relation *is a member of*, characterising such classes, is intransitive, by contrast with the relation *is a part of* which characterises sums. Suppose it is proposed that x and y are distinct coinciding distributive classes. For example, x is the class of copper atoms in region R (the region occupied by the statue); y is the class whose members are two copper arms, two copper legs, a copper head and a copper torso, all found in R . On these assumptions, x has a relation to any of the copper atoms which is incompatible with the relation of y to that atom. If z is such an atom, then z is a member of x , but not a member of y (for z is a member of a member of y , and the relation is intransitive).

Assuming, however, that both the lump of copper and the statue are sums, rather than distributive classes, no analogous incompatible (relational) properties of the superposed entities might be proposed, since the relation *is a part of* is transitive.

It appears, therefore, that although neoclassical assumptions countenance distinct superposed sums, they do not provide the resources to render such sums presently discernible from one another.

The assumption that distinct entities might be at the same place at the same time, or even superposed, at some time t , if it does not carry the further consequence that the entities are presently indiscernible, does not pose insuperable conceptual difficulties. Modern physics certainly urges us to accept that certain kinds of physical entities¹¹⁵ might be at the same place at the same time - for example, an electromagnetic field might be at the same place at the same time as a gravitational field.

¹¹⁵ Here I use 'entity' in a broader sense than the one adopted generally throughout this work, i.e. the sense of a concrete entity, where 'concrete' is understood in the way explained in Chapter 1.

If we look over a wider range of categories, we find many cases in which we accept entities¹¹⁶ being in the same place at the same time without great difficulty: two sounds, for example, may be simultaneously in one place; and on a trope-theory of universals, if a ball is red then there are distinct tropes which are in the same place (the region of space occupied by the ball) and at the same time. And as for superposition, it is not clear why it should be held to be more difficult to accept than being in the same place at the same time, at least if it is admitted that one of the sums is not identical to the entities of which it is a sum (or indeed if it is admitted that both sums have this characteristic).¹¹⁷

However, by contrast with the claims that distinct entities might be at the same place at the same time, or that they might be superposed with one another, the claim that they might be presently indiscernible at some time t does seem to present serious difficulties.

Suppose at t_1 one has identified a statue and a lump of copper which do not occupy exactly the same region of space at t_1 . It is clear that they are not identical - if for no other reason than that one has parts which the other does not. Let us call the statue ' s_1 ', and the lump ' l_1 '. Suppose at t_2 one has again identified a statue and a lump of copper, but this time the identified entities do occupy exactly the same region of space, and, moreover, they are superposed at t_2 , and indeed presently indiscernible at t_2 . Let us call the statue identified at this time ' s_2 ', and the corresponding lump ' l_2 '.

Suppose that on some grounds (say, grounds of spatio-temporal continuity) one claims that the statue identified at t_2 is identical to the statue identified at t_1 , and that the lump identified at t_2 is identical to the lump identified at t_1 . Thus one is led to assume that at t_2 there are two distinct entities which are presently indiscernible from one another, such that one of them is identical to s_1 , and the other identical to l_1 .

The difficulty that arises now is that there seems to be no non-circular answer to the question, what is it that makes it true that one of the coinciding entities rather

¹¹⁶ Again, entity is used in a wider sense than the one generally adopted in the present work (see last footnote).

¹¹⁷ In Chapter 7 I shall argue that a *classical* mereological sum is identical to all its parts. This conclusion does not extend, however, to neoclassical sums.

than the other is identical to the statue that was identified at t_1 (i.e. s_1). To see this, we first note that to say that of the two superposed entities, s_2 and l_2 , it is the one that is identical to s_1 which is the one that is identical to s_1 - is obviously circular. Secondly, to say that of the two superposed entities, s_2 and l_2 , it is the one that is identical to s^* which is the one that is identical to s_1 (where s^* is a statue identified at some time other than t_1 and t_2 , or at any time in some possible world other than the actual one) is again circular, in that a question of precisely the same type remains unanswered, namely, what is it that makes one of the superposed entities rather than the other identical to s^* .

Thirdly, and finally, to say that of the two superposed entities it is the one that is a statue which is the one that is identical to s_1 , leaves the problem equally unsolved. For given that the superposed are presently indiscernible, the property *is a statue* must be spelled out in terms of properties that are not present, and thus can be true only in virtue of an identity between one of the superposed entities and a statue identified at another time or in another possible world.

It seems then that we should accept the following as a principle governing identity statements:

Principle of Present Discernibility

PPD For all x and y , x is not identical to y if and only if
for all t , such that both x and y exist at t ,
 x is presently discernible from y at t .

Assuming that the argument above is correct, if we wish to maintain (as I think we should) that distinct wholes may be superposed with one another, we require an account of wholes other than that according to which they are conceived as sums. For, as explained above, superposed sums must be presently indiscernible, and PPD requires superposed distinct wholes to be presently discernible. If we are to have superposed distinct wholes, we must have wholes that are not sums. The notion of non-unique neoclassical *sums* is incoherent.

Section 6.3

Mereologically Varying Sums

To account for the *prima facie* fact that at least some types of wholes are susceptible to flux of parts, Peter Simons develops in some detail (as I have shown in Section 4.3.3) several versions of the notion of a sum which is expressible using a temporally modified predicate. A sum of this sort is, generally speaking, an entity which is a sum-at- t of some entities, where to say that y is a sum-at- t of the x s is to say that of all z , z overlaps y at t iff z overlaps one of the x s at t . I will now show that a certain fundamental problem arises in connection with any attempt to account for the mereological variation of a wholes in terms of such a temporally modified notion of a sum.

Let us say that if y is a sum-at- t of the x s, then y is a sum of the x s (at t). Therefore 'is a sum of ... at t ' is defined as follows:

D1 y is a sum of the x s at $t =_{\text{def.}}$

for all z , z overlaps y at t iff z overlaps one of the x s at t

The predicate 'is a sum of' is understood here as being temporally modified. Formerly in the course of the present work, 'is a sum of' was understood not as a temporally modified predicate. To allow that it should designate either a temporally modified predicate, as here, or a temporally unmodified one, as before, should not lead to confusion, however. For it should be clear from the context whether one considers a certain entity to be a sum in the temporally modified sense of the predicate or in the temporally unmodified sense.

Consider a transition which a certain whole y undergoes between two particular moments of time, t and t' . Suppose that at t y is a whole which is made up of parts $x_1 \dots x_n$ and x^* (assume, for simplicity, that none of these parts overlaps any of the others at any time). Let us abbreviate the expression ' $x_1 \dots x_n$ ' as 'the x s', and the expression ' $x_1 \dots x_n$ and x^* ' as 'the x s+ x^* '. At t , then, y is made up of the x s+ x^* . Between t and t' y loses one of its parts, x^* . The result is that at t' y is a whole which is made up of parts the x s.

To have a tangible example in mind, assume y is a house and that the $xs+x^*$ are all bricks (this is to suppose, unrealistically, that the house is made up of bricks, without mortar, beams, etc.). x^* is a brick which is knocked out of one of the walls between four o'clock (t) and five o'clock (t') in the afternoon.

The whole y changes, then, from being made up of the $xs+x^*$ at t to being a made up of the xs at t' . Accounting for the whole in terms of the notion of a sum, we assume that y is first a sum of the $xs+x^*$, and then changes into being a sum of the xs .

Two possibilities must be distinguished regarding the predicate 'is a sum of'. It is either a determinate predicate, or it is a vague predicate. If it is a determinate predicate, then at each moment of time t , and regarding any vs , it is determinately the case or not the case that u is a sum of the vs at t . If the predicate is vague, on the other hand, there may well be a time t at which it is vague whether or not u is a sum of the vs at t . Consider first the assumption that 'is a sum of' is a determinate predicate.

6.3.1 *The Assumption that 'is a sum of' is Determinate*

Since in our example y changes from being a sum of the $xs+x^*$ at t to being a sum of the xs at t' , and since the predicate is determinate, it is clear that there is some point of time at which y ceases to be a sum of the $xs+x^*$. Let us call this moment ' t_a '. Similarly, at some moment of time y begins to be a sum of the xs . Call that moment ' t_b '. Note that we are assuming that both $t \leq t_a \leq t'$ and $t \leq t_b \leq t'$. We are not as yet making any further assumption regarding the relation between t_a and t_b (i.e. on our assumptions so far, t_a might be either before, at the same time as, or after t_b). In any case, however, let us call the period of time between t_a and t_b the 'transitional period'.

Let us now look more closely at the mechanism of the transition. It would seem that the transition must take place in one of the following three ways:

- (1) The transitional period is finite (i.e. it has some positive duration), and during this period y is both a sum of the $xs+x^*$, and a sum of the xs .

(2) The transitional period is finite, and during this period y is neither a sum of the $xs + x^*$, nor a sum of the xs .

(3) The transitional period is instantaneous (i.e. it takes place in a durationless instant).

None of these options seems acceptable, however. (1) is inconsistent with the definition of 'sum'. For on (1), there is some time t in which y is both a sum of the $xs + x^*$ at t , and a sum of the xs at t . But if something is a sum of the xs at t , it cannot be a sum of the $xs + x^*$ at t (assuming as we have that x^* overlaps none of the xs at any time). For, according to D1, x^* overlaps the former sum but not the latter one.

Option (2), while not leading to a contradiction, is nevertheless similarly untenable. It allows for two possibilities.

Either

(2a) During the transitional period y is not a sum.¹¹⁸

or

(2b) During the transitional period y is a sum, but neither of the $xs + x^*$, nor of the xs .

According to (2a), if whenever y exists, y is a sum, then y does not exist during this period. However, it is obviously implausible to assume that whenever an entity loses a part, there is a transitional period during which that entity does not exist. On the other hand, if y does exist during the transitional period, it cannot be a sum during this period. The conception of y as a sum is therefore inadequate, for being a sum is not an essential feature of y .

According to (2b), y is a sum of *some* entities at any moment during the transitional period, but neither of the $xs + x^*$, nor of the xs . Now, it would seem that at any time during the transitional period, if y is a sum, then (at least) all of the xs must be among the entities of which it is a sum. That is, y is a sum of the zs , such that the xs are all among the zs (in the example case of the house this means that all of the bricks that are not affected remain parts of the house throughout the

¹¹⁸ I take the claim that u is a sum (at t) to be equivalent to the claim that for some vs , u is a sum of the vs (at t).

transition). For both immediately before and immediately after the transitional period the x s are among the entities of which y is a sum.

Furthermore, on any plausible suggestion, any entity from among the z s which is not among the x s must be a part of x^* (e.g. any part of the house during the transitional period, aside from the stable bricks, must be a part of the brick that is being knocked out). That is to say, a plausible reading of (2b) envisions a continuous process which takes place during the transitional period, where at t_a all the parts of x^* are parts of y , and gradually fewer of the parts of x^* are parts of z , until at t_b none of the parts of x^* are parts of y .

For example, when the brick is knocked out of the wall of the house, there is according to (2b) a transitional period during which the parts of the brick x^* "leak away" continuously, as it were, from being parts of the house to not being parts of the house, until, at the end of the transitional period, none of the parts of the brick are parts of the house.

This scenario is not plausible, however. For if we assume that during the transitional period the relations between the parts of x^* remain unchanged, there is no reason to think that some of the parts of x^* have ceased to be parts of y , while others are still parts of y . It would seem that the parts of x^* stand or fall together, as regards their being parts of y . Any of the (indefinitely many, in typical cases) ways of specifying *which* parts of x^* have ceased to be parts of y at any point of time during the transitional period are bound to be arbitrary, or insufficiently motivated. For example, consider the difficulty in deciding whether the first parts of the brick which should cease to be parts of the house are concentrated at the centre of the far side of the brick, or are spread evenly over the whole of the far side of the brick. I conclude, therefore, that neither (2a) nor (2b) provide an acceptable account of the loss of a part., and so option (2) is disqualified.

Turning to option (3), it is found to be similarly unacceptable. For it requires that the transition, from x^* 's being a part of y to not being a part of y , take place (and be completed) *instantaneously*, in no time at all. Now, there can be no objection to the claim that the process whereby a part of an entity is lost may be *very brief*. The twig of a tree may cease to be a part of the tree with one fell swoop of an axe. Reflection on this case reveals, however, that we do not consider the separation of the twig from the tree to be strictly instantaneous. The course of the axe through

the branch will require a fraction of a second.¹¹⁹ If our reflection progresses down to the molecular level of the process, we still find only continuous processes, which take place in non-zero periods of time (e.g. the process of distancing of the last of the attached molecules of the twig).

Admittedly, according to a controversial account of vagueness (known as the 'epistemic view')¹²⁰ instantaneous changes should be acknowledged in connection with some types of qualitative change, such as the change from an apple's being green to an apple's being red. This account acknowledges that we are unable to determine a sharp transition between being green and being red, but maintains that this is neither to be attributed to a vagueness in our associated concepts, nor to a vagueness in the properties themselves, but rather to epistemological limitations. According to this view it seems necessary to assume that the transition from being green to being red takes place instantaneously (although we are unable, in principle, to determine the point at which this transition takes place).

However, the proposal that *macroscopic quantitative* changes (i.e. changes which are not infinitesimal, nor even microscopic in magnitude), as contrasted with *qualitative* changes, might take place instantaneously is fundamentally alien both to common sense and to physical theory (whether classical or modern). But the possibility of such changes is precisely what is required on (3). If, for example, the brick weighs one kilogram, and the house (before the brick is knocked out) weighs one ton, then the weight of the house is supposed to change instantaneously by one kilogram. And the volume of the house is supposed to change in a similarly abrupt way. If a tree is chopped down, and yet the stump continues to live, the tree will have instantaneously lost several tons in weight according to the present proposal.

Finally, even if it is suggested that a future physical theory might require us to accept the possibility of instantaneous quantitative changes, it seems highly undesirable that physical theory should be constrained to this result by mereological theory. It seems reasonable to require that mereology should be able to accommodate alternative possible courses of development in physical theory in this respect.

¹¹⁹ If, for example, the axe travels at 10 m/sec, and the twig is 1cm in diameter, then it would take 1/1000 sec. for the axe to traverse the width of the twig.

¹²⁰ This account of vagueness is extensively defended in Williamson 1994.

This concludes our investigation into the proposal that the loss of a part of a whole is to be understood in terms of the assumption that the whole is a sum, where the predicate 'is a sum of' is taken to be a *determinate* predicate. Let us now look at the alternative suggestion, according to which it is assumed that 'is a sum of' is *vague*.

6.3.2 The Assumption that 'is a sum of' is Vague

If 'is a sum of' is vague, and this vagueness is attributed to the *relation* designated by the predicate (thus rejecting a linguistic or epistemic account of vagueness in this connection),¹²¹ then the following may be suggested as an account of the transitional process whereby y loses one of its parts, x^* .

Assume that the transitional period is of non-zero duration (thus avoiding the difficulties associated with instantaneous change of quantitative properties). Before t_a , y is (determinately) a sum of the $xs+x^*$. After t_b , y is (determinately) a sum of the xs . During the transitional period (between t_a and t_b), however, there are no entities such that y is (determinately) a sum of those entities. For it is neither determinately true that y is a sum of the $xs+x^*$, nor determinately false that it is their sum; and similarly, it is neither determinately true that y is a sum of the xs , nor determinately false that it is their sum.

On this account, then, on the occasion of the loss of a part, y changes from being determinately a sum of some entities (before the transitional period) to not being determinately a sum of any entities (during the transitional period), and then again to being a sum of some entities (after the transitional period). This has none of the unwelcome consequences of alternatives (1), (2) and (3) above. No contradiction is arrived at as in (1), no implausibilities regarding the intermittent

¹²¹ To assume the epistemic account would amount to repeating the proposal examined in 6.3.1; for on this account it is always determinately the case whether y is or is not a sum of the $xs+x^*$, only we are not always able to determine which is the case due to epistemic limitations. As for the linguistic account, it would seem to imply that the concept we associate with 'is a sum of' is not adequate for providing a *precise* characterisation of the relation between a whole and its parts; this seems to me to be a concession that the conception of wholes as sums is not ultimately satisfactory, which is what my criticism is meant to suggest.

existence of z as in (2a), or the piecemeal nature of the loss of y as in (2b), and no difficulties associated with instantaneous quantitative change as in (3).

However, it still seems that the notion of a sum, understood in this way, cannot be taken to be an adequate theoretical explication of the notion of a whole, in as far as this notion allows for mereological variation. For in taking y to be a whole, we take it to be determinately a whole no less before and after the loss of its part x^* , than during the transitional period in which that part is lost. It is not the case that there is no fact of the matter, during the transitional period, whether or not y is a whole. The feature of being a whole is *determinately* preserved during circumstances in which a part is lost, much like the feature of being coloured is *determinately* preserved during circumstances in which the colour of an entity is changing.

On the other hand, we have seen that during the transitional period there is indeed no fact of the matter, according to the present proposal, whether or not y is a sum.¹²² This difference points to a difference between the notion of a mereologically varying whole and a mereologically varying sum. This difference is particularly striking in the case of wholes which we consider to be continuously in a state of mereological flux, such as human organisms. An adequate account of the notion of a whole, in as much as we take this notion to apply to human organisms as paradigmatic cases, would be one which allowed us to say that under normal conditions the organism is determinately a whole. If we explicated the notion of a whole as the notion of a sum, we could not say this.¹²³

To summarize, if the notion of a sum were indeed an explication of the notion of a whole, then we should expect vagueness to be found in the application of 'whole' in precisely those cases in which vagueness is found in the application of 'sum'. Since this appears not to be the case, we have substantial grounds for

¹²² I am assuming that the claim that u is determinately a sum (at t) is equivalent to the claim that for some vs , u is a determinately a sum of the vs (at t). See footnote 117.

¹²³ Indeed, if an alternative account were not available, we might have to concede that in as far as human organisms are mereologically varying entities, they cannot be said to be determinately wholes. This might make Chisholm's mereological essentialism a more attractive position, for it denies that wholes, in the sense of 'whole' that applies to *entia per se* rather than to entities which are more properly viewed as logical constructions, are subject to mereological variation. Indeed, on this view a human organism, far from being a paradigmatic whole, is rather a mere logical construction out of entities which cannot lose parts without ceasing to exist. See Chisholm 1976, 98.

doubting that the former notion could be considered as providing an adequate account of the latter.

Chapter 7

Sums, Collections and All the Parts

In the last chapter we saw that neoclassical mereology is only partly successful in accounting for features of wholes which were not satisfactorily accounted for by classical mereology. Numerous difficulties are associated with the ways in which neoclassical mereology accounts for important features of wholes: features whereby the existence of a whole seems to depend on the way its parts are conditioned; whereby more than one whole seems capable of being made up of precisely the same entities; and whereby wholes can survive a loss or gain of parts.

The question arises, what is the source of neoclassical mereology's weakness, that is, of its inability to break loose completely from the limitations which characterise classical mereology. A clue to an answer is pursued in Section 7.1 below. Neoclassical sums inherit a certain intrinsic formal limitation from their classical ancestors. In this Section I explicate this limitation, and indicate the way this limitation renders the notion of a sum, in general, as a version of the notion of a collection which I discussed in Chapter 2.

In the Section that follows, 7.2, I argue for a connection between the notion of a sum and the feature of non-monadicity. In particular, I argue that a classical sum is to be viewed as *identical* to its parts, and therefore a classical sum is not monadic.¹²⁴ This is a controversial point, and is of interest independently of its role in my overall argument. The theory of Unities itself (which I present in the next Part) does not depend crucially on my claim that classical sums are non-monadic, though to deny this claim would be to make that theory less elegant in a certain respect. However, to the extent that my conclusion regarding the non-monadicity of

¹²⁴ Monadicity was discussed above in 1.2.6.

classical sums is acceptable, it helps to shed light from a different perspective on the distinction between sums and Unities.

Suppose it is granted that classical sums are non-monadic. Neoclassical sums are still guaranteed to be monadic, for monadicity follows straightforwardly from neoclassical assumptions embodied in the notions of a conditioned sum, a non-unique sum and a mereologically varying sum. However, the conception of a monadic entity afforded in this way is problematic, for precisely the reasons that those three particular neoclassical notions of sums were found to be problematic.

From this perspective, a connection becomes visible between the explanatory advantages of the theory of Unities, which I introduce in the next Part, and the fact that the notion of a Unity seems to offer a better grounded conception of a monadic comprising entity.

Section 7.1

A Formal Limitation Inherent in the Notion of a Sum

It is clear from our preceding discussion that classical mereology proposes what may be described as a strict correspondence between entities and sums of those entities. Whenever there exist entities, the xs , there exists, on classical principles, a unique entity y such that y is a sum of the xs . The transition from classical mereology to neoclassical mereology can be viewed as arising from an exploration of possibilities for loosening this strict correspondence. I will now show that the range of these possibilities is intrinsically limited by the definition of 'sum', and this limitation suggests that theoretical notions of a whole might be elaborated which do not share this limitation. The notion of a Unity, which I present in the next Part, is precisely such a notion.

Neoclassical mereology denies the strict correspondence laid down by classical mereology, according to which whenever there exist entities, the xs , there exists a unique entity y such that y is a sum of the xs . Instead, neoclassical mereology allows that for some xs there exists no entity y such that y is a sum of the xs . Of course, it also allows that for some xs , there exists a unique entity y , such that y is a sum of the xs . But it also allows, as a third option, that for some xs there exists a *non-unique* entity y , such that y is a sum of the xs . That is, in such a case there exists an entity z , such that $z \neq y$, and such that z too is a sum of the xs .

To summarize this contrast between neoclassical and classical mereology, let us first recall the definition of 'is a sum of':

Definition of 'is a sum of'

y is a sum of the xs =_{def.}

z overlaps y if and only if z overlaps one of the xs .

The assumptions of classical and neoclassical mereology can be contrasted as follows:

Correspondence between entities and their sums -

the classical position:

For any xs ,

for some y , y is a sum of the xs , and for all z , if z is a sum of the xs then $z = y$.

Correspondence between entities and their sums -

the neoclassical position:¹²⁵

For any xs , either

1. for all y , it is not the case that y is a sum of the xs

or

2. for some y , y is a sum of the xs , and for all z , if z is a sum of the xs , then $z = y$

or

3. for some y , y is a sum of the xs , and for some z , z is a sum of the xs , and $z \neq y$

Notwithstanding this difference between the assumptions made by classical and neoclassical mereology, we find that there is an assumption which both theories share, an assumption which imposes a certain constraint as regards the correspondence between individuals and their sums. To see this, recall the notion of coextensiveness introduced in Section 2.4:

Definition of 'are coextensive with'

the xs are coextensive with the ys =_{def.}

for all z , z overlaps one of the xs iff z overlaps one of the ys

¹²⁵ The claims I am making here apply both regarding the temporally unmodified predicate 'is a sum of' and the temporally modified one.

If one wishes to be explicit about temporal modification, one would need to add quantification over time as well. For example, the sentence

'For any xs , either 1. for all y , it is not the case that y is a sum of the xs ... '

would have to be rewritten as

'For any xs , for all t , either 1. for all y , it is not the case that y is a sum of the xs at t ... '.

For the sake of simplicity, I have left such temporal modification implicit, and provide explicitly an account involving only temporally unmodified terms.

We find that both classical and neoclassical mereology make the assumption that if u is a sum of the x s and v is a sum of the y s, then $u = v$ only if the x s are coextensive with the y s. To use terminology introduced in Section 2.4, this is to assume that sums are *coextensively determined* comprising entities.

To demonstrate that both theories make this assumption is at the same time to explain why they are compelled to do so. For as we shall see, this assumption follows from the very definition of 'is a sum of'.

The proof proceeds by supposing that u is a sum of the x s and v is a sum of the y s, and that the x s are not coextensive with the y s, and showing that it follows from this together with the definitions of 'is a sum of' and 'are coextensive with' and Leibniz's principle of indiscernibility of identicals that $u \neq v$.

Proof

- (1) the x s are not coextensive with the y s [assumption]
- (2) for some z , either
 - (2a) z overlaps at least one of the x s but none of the y s
 - or
 - (2b) z overlaps at least one of the y s but none of the x s
 [from (1) and the definition of 'are coextensive with']
- (3) (2a) [assumption]
- (4) u is a sum of the x s, and v is a sum of the y s [assumption]
- (5) z overlaps u [from (3), (4) and the definition of 'is a sum of']
- (6) z does not overlap v [from (3), (4) and the definition of 'is a sum of']
- (7) $u \neq v$ [from (5) and (6) and Leibniz's principle of indiscernibility of identicals]
- (8) (2b) [assumption]
- (9) $u \neq v$ [inference precisely analogous to that of (7)]

It follows therefore from (2), (4) and definitions and Leibniz's principle that $u \neq v$. Since (2) follows from (1) and definitions, we have the required result: it follows from (1), (4) and definitions and Leibniz's principle that $u \neq v$. And so it follows from the definition of 'is a sum of' (granted the other definition of coextensive and

Leibniz's principle) that that if the xs and the ys have the same sum, they must be coextensive.

This constitutes a constraint on the type of correspondences which may be assumed to hold between individuals and sums of those individuals, a constraint with which any theory according to which wholes are conceived of as sums must comply. This constraint expresses what might be described as a formal limitation which is inherent in the notion of a sum.

It is inherent in the definition of 'is a sum of', therefore, that sums are entities which are coextensively determined. Tracing the development of the notion of a sum from that of a collective class, we are assured that if the xs are concrete entities, a sum of the xs is a concrete, linguistically one, single entity. These features, together with that of being coextensively determined, guarantee that sums of concrete entities are collections, in the wide sense explained in Section 2.4. As we shall see in Part III, Unities are not collections in this sense. If u and v are Unities, and u comprises (i.e. is made up of) the xs , and v comprises the ys , then it may be the case that $u = v$ even in the case that xs are not coextensive with the ys .

Section 7.2

Classical Sums as Identical to their Parts

In Part I (Section 1.2) it was argued that if y is an entity which comprises the x s, the claims that the x s are many entities and that y is a single entity are compatible both with the x s' being identical to y and with the x s' not being identical to y . The supposition that the x s are many entities does not in itself compel us to think that the x s are not identical to y . This result allowed us then to assume that the comprised entities are many entities, and that the comprising entity is a single entity, while leaving open the question whether the comprising entity is identical to the comprised entities. If it is identical to them, then it is not monadic. If it is not identical to them (assuming it not identical to any other entities, the z s, if the z s are not severally identical to the x s) then it is monadic.

Having examined the notions of a classical and a neoclassical mereological sum in the foregoing discussion of the present Part, it is now of interest to return to that open question. If it is found that a whole of one type is identical to its parts, and a whole of another type is not identical to its parts, then this aspect of their respective characters offers us an illuminating way to contrast these types with one another.

I shall argue here that a classical sum is identical to its parts. In the discussion of the next Part it will become clear that a Unity can under no circumstances be identical to its parts. A neoclassical sum, typically, is also not identical to its parts. If a conditioned sum ceases to exist when its parts are dispersed, and yet those parts survive this dispersal, then clearly such a sum is not identical to its parts. Similarly, if two sums are sums of the same parts, and so they are non-unique sums, then at most one of those sums can be identical to the parts. Finally, if a mereologically varying sum survives the destruction of some of the entities that are its parts at t , then it cannot be identical to the entities that are its parts at t .

The monadicity of such sums is guaranteed precisely by means of those neoclassical mechanisms which allow the theorist to represent the conditionality of whole, their non-uniqueness, and their mereological variation. To the extent that

one is doubtful as to the success of those mechanisms, considering problems encountered by the neoclassical conception in their regard (problems of the sort raised in Chapter 6), one will also be doubtful as to whether neoclassical mereology provides us with a satisfactory conception of a monadic whole.

This point becomes significant, however, only if it is established that wholes on *some* conception are, indeed, non-monadic. For only then can it not be taken for granted that wholes are monadic, and only then does the issue of monadicity allow us to draw a contrast between different types of whole, or between different conceptions of ^awhole. I turn now precisely to this task, therefore, of establishing that wholes according to *some* conception are non-monadic - in particular, that classical sums are identical to their parts, and so non-monadic.

One terminological reminder before I proceed: in the discussion that follows I use expressions such as 'the parts', 'its part', and 'all the parts' interchangeably, to refer to entities, the *xs*, such that the whole is some *y* which is a sum of those *xs*. The contexts should make it clear (where clarity is desirable) precisely which of the whole's parts are those *xs*. By speaking of 'the parts' of a human organism, for example, one might mean molecules (where the organism is a sum of those molecules), or cells (where, again, the organism is a sum of those cells), or other parts. One does not usually (though one might) mean *both* the molecules *and* the cells. Context, as I've said, should determine which of these is intended, when something hangs on such possible differences.¹²⁶

7.2.1 *The Ambiguity*

It must first note that any claim of the form

- (1) y is identical to the *xs*.

(or equivalently 'the *xs* are identical to *y*') is ambiguous, in that it can be understood in one of two ways: *distributively*, or *collectively*. Taken distributively it is synonymous with

¹²⁶ See in this connection the remarks on 'the parts' in Chapter 3 (3.1.4.4).

- (1a) y is identical to *each* of the x s.

Taken collectively, it is synonymous with

- (1b) y is identical to *all* of the x s *taken together*.

The phrase 'taken together', in the latter sentence, is of course metaphorical. To explain its significance, and the significance of the contrast between the two readings of ' y is identical to the x s', we may consider an analogous contrast between different readings of sentences in which the predicate is not 'is identical to' (or 'are identical to') but rather some predicate of the form 'is identical in respect of A '. Consider, for example, the sentence

- (2) The people are identical in (i.e. in respect of) weight to the stone

This sentence can be read in two ways. Read in one way, it is synonymous with

- (2a) Each of the people is identical in weight to the stone

Read in another way, it is synonymous with

- (2b) All the people taken together are identical in weight to the stone

The latter sentence (2b) is understood as implying that the (arithmetical) sum of the people's weights is identical to the weight of the stone. The former sentence (2a) clearly does not imply this, for in case that the number of people is greater than one (2a) is not even compatible with this implication. Therefore it is clear that the two readings are not equivalent to one another.

To claim that ' y is identical to the x s' can be taken either distributively or collectively is to claim that this sentence is susceptible to alternative readings which contrast in a way analogous to the two readings of the sentence 'the people are identical in weight to the stone'. The analogy also leads us to assume that the two readings of the former sentence, (1a) and (1b), are not equivalent to one another. Let us say that the statement ' y is identical to the x s', taken distributively, is a *distributive identity* statement, while the statement, taken collectively, is a *collective identity* statement. To distinguish the two readings I will use ' y is distributively identical to the x s' when distributive identity is meant, and ' y is collectively identical to the x s' when collective identity is meant.

It would seem wrong to interpret contrast between distributive and collective identity statements in terms of a distinction between *kinds* of identity, or *senses* of 'identity'. Compare this with the claim that Samson lifted two stones, which might be understood as the claim that he lifted the stones either at different times, or at the same time. It would be strange to assume that this is to distinguish between two kinds of lifting, or two senses of 'lift'.

It would be more plausible to suggest that the distinction between the two types of statement hinges on the question of which entities are asserted to stand in the relation *is identical to* to one another. There is a unique relation *is identical to*, but in a distributive identity statement *y* is asserted to stand in this relation to each of the *xs*, while in a collective identity statement this is not asserted, but rather that *y* stands in this relation to all of them collectively.

Paralleling the two interpretations of (1), we have two corresponding interpretations of sentences of the following form:

(3) *y* is distinct from the *xs*.

Namely,

(3a) *y* is distinct from each of the *xs*.

(3b) *y* is distinct from all the *xs* taken together.

In the case of (3a) we will say that *y* is *distributively distinct* from the *xs*, while in the case of (3b) we will say that *y* is *collectively distinct* from the *xs*. Note that whereas (1b) and (3b) are contradictories, (1a) and (3a) are merely contraries.

7.2.2 *An Objection on Categorical Grounds*

Commenting on what Doepke calls 'The One-Many View', Simons notes that this view

disallows (quite properly) identity between a plurality of objects, such as a number of wooden boards, and a single object, such as a ship, made up of them.¹²⁷

Both Doepke and Simons deny that this view provides good reasons for rejecting Wiggins's conclusion that distinct entities may be superposed with one another.¹²⁸ However, they both seem to think that the principle on which the response of upholders of the One-Many View is based is true. Thus van Inwagen, Doepke, Simons, and anyone who subscribes to the One-Many View, are apparently all in agreement that statements of the form (1) should be "disallowed", unless the *xs* are all identical to one another.

Now, it is clear that (1a) can only be true if the *xs* are all identical to one another. Denying this would lead immediately to a contradiction. For suppose *a* and *b* are among the *xs*, such that $a \neq b$. From (1a) it follows that $y = a$ and that $y = b$. Assuming that identity is symmetric and transitive, this implies that $a = b$, contradicting the assumption that $a \neq b$.

However, it seems that no contradiction can be derived from (1b). Presumably the reason for disallowing (1b), in case the *xs* are not all identical to one another, would then be that this statement involves some sort of category mistake; that it is in some way nonsensical.

It is not clear, however, why (1b) should be thought non-sensical in such non-trivial cases (i.e. cases in which the *xs* are not all identical to one another). Perhaps the reason for thinking this is that it is commonly supposed that *is identical to* is a two-place relation.¹²⁹ Certainly this is supposed with regard to the relation as it is understood in the context of standard formal logic (i.e. predicate calculus with identity). If *is identical to* is a two-place relation then to claim, for example, that

¹²⁷ Simons 1987, 212.

¹²⁸ See van Inwagen 1990, 287 (note 13); Doepke 1982, 46-7; Simons 1987, 212.

¹²⁹ See, for example, Russell 1903, 95-6; Grossmann 1983, 170-2.

distinct entities x and y are collectively identical to an entity z is to fail to understand the predicate 'is identical to', taking it to designate a three-place relation, or a multigrade relation, instead of a two-place one. A mistake would be involved here which is similar to the mistake involved in claiming that x asks y , while denying that there is some z such that z is what x asks y . In this latter example, the mistake would consist in taking a triadic predicate ('asks') to be a dyadic predicate.

The fact that *is identical to* is understood as a two-place relation in the context of standard logic does not, however, provide sufficient evidence for the view that statements of the form (1b), in non-trivial cases, are nonsensical. For it is widely recognized that standard logic is incapable of representing the full range of valid inferences made in common discourse. And the question of how standard logic must be supplemented in order to represent adequately the inferences of common discourse is still far from settled.¹³⁰ Just as it is appropriate to inquire whether the reduction of plural quantification to singular quantification is generally justified, so it would seem appropriate to inquire whether the reduction of collective identity to distributive identity is generally justified.

On the positive side, Donald Baxter has helpfully pointed out that common sense judgements often seem to betray the assumption that statements of the form (1b) in many cases not only make sense, but are true. He remarks plausibly as follows:

Someone with a six-pack of orange juice may reflect on how many items he has when entering a 'six items or less' line in a grocery store. He may think he has one item, or six, but he would be astonished if the cashier said 'Go to the next line please, you have seven items'. We ordinarily do not think of a six-pack as seven items, six parts plus one whole.¹³¹

It is a common sense view that in this case we have six bottles of orange juice, and that at the same time we have one six-pack of orange juice, and yet we do not have seven items. Rather, we have either six items or a single item, depending, one might suggest, on the way we wish to describe what we have. The various

¹³⁰ On this question, see Boolos 1984.

¹³¹ Baxter 1988, 579.

judgements involved here are compatible if, and only if, the six items are identical to the single item. *Prima facie*, then, it would seem that in some cases it is true to claim that *y* is collectively identical to the *xs*.

7.2.3 *Arguments Purporting to Refute the Thesis*

Assuming, then, that statements of the form (1b) are acceptable at least from the categorial perspective (that is, assuming that they make sense), there seems to be a very strong case indeed, *prima facie*, for claiming that a classical sum is collectively identical to its parts. First of all, this is suggested by the fact that the sum and the parts (collectively) have precisely the same identity conditions. If *y* is the (classical) sum of the *xs* then *y* exists if and only if the *xs* exist.

Secondly, it is suggested by the fact that the sum occupies at all times precisely the same region of space as the parts (collectively) do. Indeed, if a single entity *x* (rather than many entities, the *xs*) stood in such relations to *y*, there would hardly be any doubt in our minds that *y* is identical to *x*. It seems to me that these considerations clearly place the burden of proof on those who would argue that the sum is *not* identical to its parts.

Therefore, I propose in what follows to consider several arguments which purport to show that a sum is not identical to its parts. To the extent that the answers that I offer in response to these arguments are satisfactory, I shall consider the claim that the sum is identical to its parts to have been established, pending the presentation of new reasons for denying this claim.

I shall begin by discussing an argument which Donald Baxter considers to be the principal argument against the view that a sum is identical to its parts.¹³²

¹³² See Baxter 1988. Throughout his discussion, Baxter uses the term 'whole', rather than 'sum' or 'classical sum'. As I have noted above, however, there are straightforward reasons for thinking that a neoclassical sum is *not* identical to its parts. The reasons he gives, both for and against the view that the whole is identical to its parts, are at their most plausible if phrased with respect to the particular type of whole we have been calling 'classical sums'. To keep this clearly in mind, I have substituted his 'whole' for 'sum' throughout, where I take 'sum' as short for 'classical sum'.

Baxter contrasts two views that one might take regarding sums, the Identity view (which I call 'SIP') and the Non-Identity view (which I call 'SDP'):

SIP The sum is identical to all its parts.

SDP The sum is distinct from all its parts.

Position SIP, he claims, is the one commonly taken by people who are not professional philosophers. By contrast, in professional philosophical contexts the more commonly held position is the SDP.¹³³

Baxter thinks that this divergence between common sense and professional opinions is to be explained by the fact that philosophers are confronted with the following argument (which I call 'Argument I') against SIP, which is "straightforward and compelling":¹³⁴

7.2.3.1 *Argument I*

The sum and all the parts exist. The sum comprises all the parts. None of the parts do. So the sum has a (relational) property none of the parts has. So the sum is identical with none of the parts. So it is distinct from each. So if n parts exist and the sum exists, then (at least) $n+1$ things exist.¹³⁵

The first three lines of this argument are concerned with establishing that the sum is distinct from *each* of its parts. Argument I consists, then, of the following three steps

¹³³ Baxter 1988, 578. Of course, people who are not professional philosophers are more likely to use the term 'whole' than 'sum' (indeed, they are not likely to debate about classical mereological sums). However, Baxter's point is that in common sense thinking about wholes *in general* there is an inclination to consider the whole to be identical to its parts. Thus if common sense intuition is to be represented in a debate about classical sums, this would reasonably be done by attributing to common sense (according to Baxter) the belief that a sum is identical to its parts (SDP).

¹³⁴ Baxter 1988, 578.

¹³⁵ Baxter 1988, 578-9, substituting 'sum' for 'whole'.

- (i) The sum is distinct from each of the parts.
- (ii) If n parts exist and the sum exists, then (at least) $n+1$ things exist.

Therefore,

- (iii) The sum is collectively distinct from its parts

Baxter first considers a response to this argument which he finds unsatisfactory.¹³⁶ He then presents his own response, according to which it is denied that (ii) follows from (i). And since it is presumably clear that (iii) does not follow from (i) on its own, this blocks the inference to (iii).

This proposal involves denying that the sum is something in addition to its parts (i.e. denying (ii)), as a means for upholding the claim that the sum is collectively identical to the parts. He thinks that such a proposal can be justified by appealing to Butler's view of Identity:

Assume that on strict standards for counting the parts are many and on loose standards they are one. The strictly distinct parts are identical with each other

¹³⁶ This response expresses what he calls the 'Combination' view, according to which

... if there are six things then a seventh thing - the sum they are parts of - exist, and the six things collectively are the seventh thing. [Baxter 1988, 579; as before, substituting 'sum' for 'whole']

He explains that the Combined view involves the assumption that a sum is a *multitude*, rather than a *single thing*, where it is assumed, nevertheless that a multitude is *one thing* no less than a single thing is. Baxter apparently takes it that the sum's being one thing would explain why it counts as a *seventh* thing, while its being a multitude (rather than a single thing) would explain why this is compatible with its being collectively identical to the parts.

I have to admit that I find this response obscure. In the first place, I don't understand his contrast between a multitude and a single thing. The multitude is asserted to be one thing, and yet I cannot make out a sense of 'being one' which would suit the argument's purposes. In the second place, even if a consistent sense could be given to the terms he uses, the argument does nothing in the way of explaining *how* it is that if an entity counts as a seventh thing, that is as something in *addition* to six things, it may nevertheless be (collectively) identical to the six things. This seems to ignore the widely acknowledged close conceptual connection between counting and identity.

However, as I have noted, Baxter himself finds this response unsatisfactory, although for different reasons. His main reason seems to be that the Combination view entails that all sums are multitudes, and this, he claims, commits us to atomism, and "one's theory of parts and whole should not mandate atomism". However, his response suffers from the unclarities noted with regard to the notion of a multitude, no less than does the Combination view itself. See Baxter 1988, 580.

on a loose standard. The whole, then, is just the parts counted loosely. It is strictly a multitude and loosely a single thing.¹³⁷

The example by means of which Baxter illustrates the distinction between counting on strict standards and counting on loose standards involves a sapling and the mature oak tree into which it develops in the passage of time. The sapling and the mature oak are taken to be strictly distinct temporal parts of the temporally extended oak tree.

Counting according to strict standards (according to Butler) the sapling is one thing and the mature tree is another. Counting according to loose standards the sapling is one thing and the mature tree is that one thing again.¹³⁸

Baxter's treatment of Argument I requires us to apply a similar distinction in connection with counting the (contemporaneous) parts of a sum. Suppose x and y are what we normally take to be distinct parts of a sum (say, two of the orange juice bottles, which are distinct parts of the sum of six such bottles). Then, counting on strict standards, x is one thing and y is another. Counting according to loose standards, x is one thing and y is that one thing again.

The resulting picture is as follows: Counting strictly there are n entities. Counting loosely, there is one entity. Under neither standard of counting do we arrive at $n + 1$ entities. Thus assumption (ii) is denied.

I find this solution unconvincing. The notion of a loose identity as required by Baxter is a strange one. For on his account, entities x and y are loosely identical to one another if, and only if, there is some entity z of which they are both parts (see penultimate quote above). Thus, for example, if the Earth is a sum, and Mount Everest and Loch Lomond are parts of the Earth, then Mount Everest and Loch Lomond are supposed to be loosely identical to one another. But clearly the relations between them bear little if any analogy to the relation *is identical to*, so that the use of the predicate 'is loosely identical to' would not be appropriate with regard to them, unless in virtue of a completely altered meaning of 'identical' as featuring in that predicate. But if Baxter uses 'identical' in a radically altered sense,

¹³⁷ Baxter 1988, 580. With regard to Baxter's notion of a *multitude*, see last footnote.

¹³⁸ Baxter 1988, 576-7.

the assumptions which his argument requires, connecting the notion of identity with the notion of counting (for he says that since the parts are loosely identical, they count on loose standards as *one* entity), seem to be unfounded.

Indeed, one would be forgiven for thinking that what Baxter means by 'loosely identical to' is simply 'part of the same sum as'. But in itself, the fact that the relation *is part of the same sum as* holds between the parts of a sum gives us no reason to think that the sum is collectively identical to its parts.

I would suggest that Baxter's account is intuitively grounded in the undeniable observation that there is a certain sense in which we might speak of many entities (even if radically distinct from one another) "as one". This, however, should in no way incline us to suppose that in such cases we treat the many entities as "loosely identical to one another". Rather, I think that the correct account of what we mean in such cases is as follows: Instead of simply speaking of the many entities, we speak in such cases of a single entity that purportedly comprises those entities. To describe speaking of the comprising entity as a way of speaking about the many comprised entities is perhaps acceptable even in cases where the comprising entity is deemed not to be identical to the comprised entities, though regarding such a case the description would indeed be loose. But as I said, the looseness here does not concern a supposed identity of the comprised entities to one another.

Notwithstanding my rejection of Baxter's response, however, I think that a plausible account is available by reference to which the inference of Argument I can be blocked.

Argument I is based on certain presuppositions regarding the relation between judgements as regards number (i.e. counting) and judgements of distinctness. Now, while it could hardly be denied that there is some such relation between the two types of judgement, I will show that Argument I is based on an inconsistent account of this relation.

Consider the second step of Argument I. It can be seen by inspection that the derivation of (ii) from (i) presupposes the following principle relating distinctness and counting:

A if the x s are n in number then

if y is distributively distinct from the x s, then the x s+ y are $n+1$ in number

Principle A is plausible. An example: if those bottles are 5 in number then - if this bottle is distributively distinct from those bottles (i.e. distinct from each of those bottles), then those bottles and this are 6 in number. Another example: if those bottles are 6 in number then - if this pack (i.e. a six-pack of bottles) is distributively distinct from those bottles, then the bottles and the pack are 7 items in number.

The converse of A is equally plausible. It would make little sense to assume that those bottles are 5 in number, and this bottle is distributively distinct from those, and yet those bottles and this are not 6 in number. Therefore, A is reasonably viewed as a partial statement of the following more complete principle:

A* if the x s are n in number then

y is distributively distinct from the x s iff the x s+ y are $n+1$ in number

Next, consider the second step of Argument I. Again inspection reveals that the derivation of (iii) from (ii) presupposes the following principle relating distinctness and counting:

B if the x s are n in number then

if the x s+ y are $n+1$ in number then y is collectively distinct from the x s

Principle B is also plausible. If those bottles are 5 in number then - if those bottles and this one are 6 in number, it follows that this bottle is collectively distinct from those bottles (i.e., this bottle is distinct from those bottles taken together). Similarly, if the bottles are 6 in number then - if the pack and the bottles are 7 items in number then the pack is collectively distinct from the bottles (i.e., again, the pack is distinct not only from *each* of the bottles, but is also distinct from all of them taken together).

The converse of B, however, is not plausible - indeed, it is clearly false. According to the converse of B, supposing b_1 , b_2 and b_3 are three bottles, and bottle b_3 is collectively distinct from b_1 , b_2 and b_3 (for b_3 is distinct from b_1 , b_2 and b_3 taken together), it follows that b_3 and b_1 , b_2 and b_3 are 4 in number. But this is false.

This suggests that a more complete principle is presupposed in the derivation of (iii) from (ii), a principle that implies B, and whose converse is as plausible:

B' if the x s are n in number then
 if the x s+ y are $n+1$ in number then y is collectively *and* distributively
 distinct from the x s

Both B' and its converse seem unobjectionable, as experimenting with examples of bottles and packs quickly shows. Combining the B' with its converse, we arrive at the following principle:

B* if the x s are n in number then
 y is collectively *and* distributively distinct from the x s iff
 the x s+ y are $n+1$ in number

Comparing A* and B* we see that Argument I makes use of two distinct principles regarding the connection between distinctness and counting. Significantly, to assume that the two principles are compatible with one another is to assume that if y is distributively distinct from the x s then it is also collectively distinct from the x s. But this is precisely what Argument I sets out to establish. So that in making use of *both* principles, Argument I is either simply begging the question, or guilty of inconsistency. For on inspection it is clear that if one consistently used either of these principles in Argument I, step (iii) could not be derived. If (A*) were used, then step (iii) could not be derived from (ii). If (B*) were used, then step (ii) could not be derived from (i). We may conclude then that Argument I must be rejected.

As to the question, which, indeed, of the two principles A* and B* is the correct one, I think this is most plausibly viewed as a matter of convention.¹³⁹ In the context of ordinary everyday counting, both yield the same results. Suppose we are counting the oranges in a plastic bag, pointing to them consecutively. Suppose we have already counted five of them. Pointing now to an orange, we may well be in doubt as to whether it is identical to one of those already counted. That is, we may well be in doubt about whether this orange is distributively distinct from the

¹³⁹ In fact, another plausible principle may be proposed, replacing both A* and B*:

C* if the x s are n in number then
 y is collectively and distributively distinct from the x s and
 for any z , if the z s are among the x s then
 y is collectively distinct from the z s
 iff
 the x s+ y are $n+1$ in number

oranges already counted. We are always clear, however, that the orange is collectively distinct from the oranges already counted.

In the more unusual cases where the different principles yield different results, when one is asked how many entities there are here, it is appropriate to reply: 'how many on which counting convention?'. Returning to the case of the six bottles and the six-pack of those bottles, according to A* one would say that there are seven (whether or not the sum is collectively identical to its parts); according to B* one would say either that there are six (assuming that the sum is collectively identical to its parts), or that there are seven (assuming that the sum is collectively distinct from its parts).

David Lewis supports a view which comes very close to affirming SIP, and takes his own view to be sympathetic, and similar, to that of Baxter.¹⁴⁰ He calls his position on this matter the thesis of Composition as Identity. He too, however, is not convinced by Baxter's argument, and as a result makes his position out to be somewhat weaker than unqualified embracing of SIP (and so weaker than the position I am arguing for). He claims that mereological relations, such as *compose*, *is a part of*, and *overlaps*, are

strikingly analogous to ordinary identity, the one-one relation that each thing bears to itself and to nothing else. So striking is this analogy that it is appropriate to mark it by speaking of mereological relations ... as kinds of identity. Ordinary identity is the special, limiting case of identity in the broadened case.¹⁴¹

With regard to the relation between a single part and its sum (assuming that the sum has parts which are not identical to that part), the claim for an analogy with identity is related to David Armstrong's position, according to which Australia and New South Wales, for example,

are not identical, but they are not completely distinct from each other. They are partially identical.¹⁴²

¹⁴⁰ Lewis 1991, 81-7.

¹⁴¹ Lewis 1991, 84-5.

¹⁴² Armstrong 1978, Vol. II, 37-8.

It is of course clear that the relation between a single part and its sum can at most be taken to be analogous to identity. One would hardly wish to argue that a single part is literally identical to its sum (assuming, again, that the sum has parts which are not identical to that single part).

Regarding the relation between *all* the parts and their sum, Lewis interestingly points out several ways in which it is analogous to identity, ways which he regards as very strong evidence for the conceptual proximity between composition and identity.¹

Why then does Lewis refrain from simply embracing SIP? it is important for him, after all, to assert that (classical) mereology is "ontologically innocent", that is, that if one starts by commitment to the existence of several individuals, and one adopts the assumption that a sum of those individuals exists, one does not thereby increase one's original commitment. And this assertion would be made more compelling if it could be claimed that a mereological sum is identical to all its parts (rather than merely related to all its parts in a way analogous to identity).²

He presents two arguments for rejecting this otherwise desirable view. Neither of them, however, seems to me to be convincing. I shall now examine them in turn, dubbing them Argument II and Argument III, respectively.

7.2.3.2 *Argument II*

In the first place, I know of no way to generalize the definition of ordinary one-one identity in terms of plural quantification. We know that x and y are identical iff, whenever there are some things, x is one of them iff y is one of them. But if y is the sum of the x s, then there are some things such that each of

¹ See Lewis 1991, 84-87.

² Simons takes Lewis's ultimate rejection of the claim that the sum is identical - strictly speaking - to its parts, to be incompatible with Lewis's earlier claim that "given a prior commitment to cats, say, a commitment to cat-fusions is not a *further* commitment" (Lewis 1991, 81). See Simons 1991, 396.

the *xs* is one of them and *y* is not; and there are some things such that *y* is one of them but none of the *xs* is.³

By way of considering this argument, I note first with regard to Lewis's formulation of 'the definition of ordinary one-one identity' as follows: to say that there are some things such that each of the *xs* is one of them and *y* is not, is equivalent to saying that each of the *xs* has a property (non-relational or relational) which *y* does not have. For instance, to say that there are sheep in the valley such that each of the black sheep in the valley is one of them and the sum of the black sheep is not, is equivalent to saying that each of the black sheep in the valley has a property - namely, *is a black sheep and is in the valley* - which the sum of the black sheep does not have.

Lewis is right, of course, in pointing out that if *y* is a sum of the *xs*, then each of the *xs* has a property that *y* does not have, and similarly that *y* has a property that each of the *xs* does not have. For example, since in cases of the type under consideration each of the *xs* is assumed to be a proper part of *y*, each of the *xs* has the (relational) property *is a proper part of y*, whereas *y* does not have this property. And *y* has the property *has each of the xs as a proper part*, whereas none of the *xs* has this property.

All that can be said on the basis of the 'definition of ordinary one-one identity', however, is that these observations about sums and their parts imply that *y* (i.e. the sum of the *xs*) is different from *each* of the *xs*. Using the terminology I introduced earlier (in 7.2.1), this is to say that *y* is *distributively* distinct from the *xs*. The thesis under examination, however, does not deny this, but states that *y* is *collectively* identical to the *xs*. Furthermore, Argument II gives us no reason for thinking that the distributive distinctness of *y* from the *xs* entails that *y* is not collectively identical to the *xs*.

Lewis's argument is well taken, in as far as it shows that 'the definition of ordinary one-one identity' does not provide us with the resources for establishing that the sum is collectively identical to the parts. In itself, however, this should not incline us to doubt that the sum is collectively identical to the parts, since the identity involved here is *not* one-one, but one-many. Presumably Lewis accepts

³ Lewis 1991, 87; substituting 'sum' for 'fusion'.

this, and therefore his qualms must be rooted in his doubt that the 'definition' might be generalised to the one-many case. Perhaps if such a generalisation is unattainable it may be reasonable to hold, as Lewis does, that the relation between a sum and its parts is merely *analogous* to identity.

Lewis does not provide us with any substantial reasons for doubt regarding the prospects of such a generalisation, however. Merely to declare (as he does) that one does not know how to carry out such a generalisation is not, of course, to give a principled reason for supposing that the generalisation is unattainable, and so would seem to constitute a very weak point against the claim that the sum is collectively identical to its parts.

This is so especially because proposals for generalisation which are at least apparently promising are available, proposals which Lewis does not take note of. For example, consider a 'definition of one-many identity' as follows: y and the x s are (collectively) identical iff, whenever there are some things, y is one of them iff the x s are one of them. I shall not explore this, nor other possible proposals, and to that extent I am perhaps leaving unresolved a certain doubt as to whether the sum can be considered as collectively identical to its parts. As I noted above, however, it would seem appropriate that in connection with this claim the burden of proof should lie on those who wish to reject it, given the *prima facie* reasons in its support.

7.2.3.3 *Argument III*

And in the second place, even though the many and the one are the same portion of Reality, and the character of that portion is given once and for all whether we take it as many or take it as one, still we do not really have a generalized principle of indiscernibility of identicals. It does matter how you slice it - not to the character of what's described, of course, but to the form of the description. What's true of the many is not exactly what's true of the one. After all they are many while it is one. The number of the many is six, as it might be, whereas the number of the sum is one. And the singletons of the many parts are wholly distinct from the singleton of the one sum.¹⁴⁶

¹⁴⁶ Lewis 1991, 87; again, substituting 'sum' for 'fusion'.

Lewis is claiming here that whenever y is a sum of the x s, certain predicates apply to y which do not apply to the x s, and conversely certain predicates apply to the x s which do not apply to y . If this is the case, then indeed to claim that y is collectively identical to the x s would be to violate the principle of indiscernibility of identicals, and therefore this latter claim should be rejected.

Let us consider his position more carefully. Let our example case be that of six horses and their sum. Lewis maintains that these differ because (1) the horses are many while the sum is one; (2) the number of the horses is six while the number of the sum is one; and (3) the singletons of the horses are "wholly distinct" from the singleton of the one sum. Point (3) is very different from the first two points, and may be dealt with straightforwardly.

It seems that in the case of (3) we have again a confusion of the sort that was noted in Argument II. Granted, u and v cannot be identical unless they have the same singletons. And granted, the sum of horses has a singleton which is distinct from the singletons of each of the horses. This only shows, however, that the sum is distinct from each of the six horses, that it is *distributively* distinct from the horses. This is hardly a difficult claim to accept, on any plausible view. But I have already pointed out several times that distributive distinctness does not imply collective distinctness. On the contrary, had the sum of horses not been distributively distinct from the six horses, it would be impossible for it to be collectively identical to them. Lewis's point (3), then, gives us no reason at all to doubt that the horses can be collectively identical to the sum.

Points (1) and (2) do not involve a confusion of this sort. It is not completely clear what Lewis thinks the conflict is, exactly. Presumably regarding (1) he takes it that the six horses are many while the sum *is not* many; and that the sum is one, whereas the horses are not one. It is by no means clear, however, why the sum is supposed not to be many, and why the six horses are supposed not to be one. To make such claims would not seem to put to great a strain on linguistic usage. Indeed, what is known as the 'constitutive' use of the copula involves precisely such statements, at least when oblique statements are used: 'the many molecules are (i.e. constitute/compose/make up) one table' and 'the table is (i.e. is constituted/composed/made up by) many molecules'; and hence, obliquely, 'the many molecules are one' and 'the table is many'.

Perhaps Lewis takes (1) to be simply making the same point as (2), only somewhat differently formulated. Now, in connection with (2) it *does* seem that a sentence like 'the number of the sum is six' would be seriously at odds with accepted usage, by contrast with 'the number of the horses is six'. This, however, is still far from showing that the horses are not collectively identical to the sum. Consider the analogous question, whether it is possible for a man to be identical to a woman. Surely this question cannot be settled by the merely linguistic consideration, according to which a sentence such as 'he talked to herself' is at odds with usage (perhaps the question is conceptual; perhaps empirical).

Perhaps the most natural interpretation of the points Lewis is making in (1) and (2) is not one that takes them rely on linguistic considerations. According to this interpretation, the claim that the horses are many, for example, means the same as the claim that the horses are (i.e. are identical to) many horses. And the claim that the sum is one means the same as the claim that the sum is (i.e. is identical to) one sum. Similarly, the claim that the number of the many is six means the same as the claim that the many are six horses; and the claim that the number of the sum is one means the same as the claim that the sum is one sum.

If this is how Lewis's claims are to be taken, however, it is not clear on what grounds he is denying that what is true of the many is also true of their sum. If the horses are (i.e. are identical to) many horses, it is not clear on what basis it is being urged that the sum of horses is not also (i.e. is not also identical to) many horses. And if the sum is (i.e. is identical to) one sum, it is not clear on what basis it is being denied that the horses are (i.e. are identical to) one sum.

The claims in the last paragraph do not seem to involve linguistic impropriety (see my comments several paragraphs above concerning the constitutive use of the copula). To reject them as false because of the principle that many entities cannot be identical to one, or in particular that many parts cannot be collectively identical to their sum, is of course to beg the question. For Lewis's argument attempts to lead precisely to that principle as its conclusion.

Therefore, as far as I can see, Lewis's discussion gives us no substantial reasons for thinking that a sum cannot be collectively identical to its parts. All in all, the evidence we have found for the assumption that a sum is collectively identical to its parts well outweighs the evidence against it.

III Unities

Chapter 8

A Theory of Unities

Section 8.1

Introduction

In Section 7.1 we observed that a certain constraint follows from the way the notion of a sum is defined, with regard to the types of correspondences allowed between individuals and their sums. Thus we found that if the x s and the y s have the same sum (at t), they must be coextensive (at t). This constraint provides us with a clue as to a way in which an account of wholes might be given according to which they are not construed as sums. To see this, however, we need to take a step back and examine the notion of a whole with the view to explaining, rather than simply taking for granted, its alleged connection with the notion of a sum.

Before embarking on this in the next Section, however, I would like to give an overview of the idea I shall be pursuing, for fear of the many trees blocking the view of the forest later, when we enter various details. I shall describe, therefore, the gist of the picture, postponing the detailed justification for the suggestions until such a first sketch is already before us.

We start by noting again that when we say that certain parts *make up* a whole, we mean more than that, simply, each of them is a part of the whole. Indeed, the

claim that they make up the whole entails that each of them is a part of the whole. However, the claim that each of them is a part of the whole does not entail that they make up the whole. My fingers are parts of my body, but they do not make up my body.

This indicates that between wholes and their parts two relations must be distinguished. One is the relation which a whole has to its part *distributively*, that is, a relation which consists in *each* of those parts bearing the relation *is a part of* to the whole. The other is the relation which a whole has to its parts *collectively*, a relation consisting in those parts - considered *together* - bearing the relation *make up* to the whole. I distinguish, therefore, between two 'whole-parts' relations: the distributive whole-parts relation, which I call 'is a whole with respect to'; and the collective whole-parts relation which I call 'is a whole which corresponds to'.

The conception of wholes as sums is founded on the assumption that the collective whole-parts relation is no other than the relation *is a sum of*. This involves, as we have seen, the assumption that wholes are coextensively determined. I describe this as the view that wholes are *fundamentally* sums. According to this view, therefore, *y* is a whole which corresponds to the *xs* only if *y* is a sum of those *xs*. It is remarkable, however, that this assumption is not a consequence of the principles adopted^{by} neoclassical mereology (nor even those of classical mereology). One may assume that the relation *is a proper part of* is irreflexive and transitive, and that it complies with the principle Weak Supplementation, and even with the classical assumption connecting *is a part of* with *is a proper part of* and *is identical to* - and yet consistently assume that something is a whole which corresponds to the *xs* without being a sum of whose *xs*.

A theory may be proposed, therefore, which accepts the latter fundamental mereological assumptions, and yet rejects the assumption that the collective whole-parts relation is necessarily the relation *is a sum of*. Indeed, it rejects the assumption that all wholes are coextensively determined. According to this theory, a Unity is an entity which may well be a whole which corresponds to the *xs*, without being a sum of the *xs*. A Unity, therefore, is a whole which is *not* fundamentally a sum. It is still the case that a Unity is a sum of *some xs*, however. This is what I mean by saying that it is *peripherally* a sum. It is a whole which corresponds to those *xs* and at the same time to some *ys*, where it is a sum of the *xs* but not a sum of the *ys*.

Suppose, for example, a tree has two main branches, Branch1 and Branch2. Let us assume, then, that it is made up (at some time t) of Root, Trunk, Branch1 and Branch 2. According to the conception of wholes as sums, this implies that the tree is *not* made up (at t) of Root, Trunk and Branch1. Anything that is a sum of the latter cannot be identical to something that is the sum of Root, Trunk, Branch1 *and* Branch2. For the parts which exclude Branch2 are not coextensive with the latter parts, which include Branch2.

According to the conception of wholes as Unities, by contrast, it may well be the case both that Root, Trunk, Branch1 and Branch2 make up the tree (at t), and that Root, Trunk and Branch 1 make up the tree (at t). Unities are not assumed^d to be coextensively determined, and a Unity may be a whole which corresponds to certain parts without being a sum of those parts. Indeed, to assume that the tree is a Unity is not to deny that it is *peripherally* a sum. The tree, in this case, is clearly a sum of Root, Trunk, Branch1 and Branch2. However, the tree is not fundamentally a sum, and so it can be made up also by parts of which it is not a sum.

I shall be assuming that whenever the xs make up a Unity, and each of the xs is an *immediate* proper part of the Unity (i.e. it is not a proper part of any proper part of the Unity), then the xs *underlie* the Unity. The relation *underlie*, then, is the relation which immediate proper parts - taken collectively - have to the whole, if that whole corresponds to them. An immediate proper part of a Unity is called an *element* of the Unity. A proper part of a Unity, in general (i.e. whether immediate or not), is called a *pre-element* of the Unity. The version of the theory of Unities which I propose here is developed in terms of assumptions about the relations *underlie*, *is an element of*, and *is a pre-element of*.

A more comprehensive account would treat not only *underlie*, but also other types of the relation *make up* which apply in connection with Unities - that is, types which apply not only between a Unity and its *immediate* proper parts. However, there are many details which have to be attended to even if one confines oneself to a discussion of *underlie*, and so I consider it appropriate to confine the discussion in this way, given that the present work only aims to put forward points *towards* a Platonic theory of wholes and parts, rather than pretending to exhaust the issues raised.

Assuming, for example, that Root, Trunk, Branch1 and Branch2 are *immediate* proper parts of the tree, then they will be said to be elements of the tree, and (taken collectively) to underlie it. To assume that these elements underlie the tree is consistent with assuming that Root, Trunk and Branch1 (without Branch2) underlie the tree; or indeed with assuming that Root, Trunk and Branch2 (without Branch1) underlie the tree. At the same time it is consistent with assuming that Root and Trunk (without both Branch1 and Branch2) are elements of the tree which *do not* underlie it.

If Root, Trunk, Branch1 and Branch2 are elements of a Unity which is the tree, then none of the molecules which are parts of either of those elements can be an element of the tree. For the molecules are proper parts of proper parts of the tree, and so are pre-elements, but not elements, of the tree. As a consequence, no molecules are such as to underlie the tree. In particular, it is not even the case that *all* the molecules which are parts of the tree at some time *t* (ignoring for the moment the possibility that some molecules are *vaguely* parts of the tree) underlie the tree at that time.

It is consistent with the assumptions that I will be making, however, that those molecules do underlie some other entity which is distinct from the tree - perhaps a lump of wood. For this reason, the theory of Unities as I develop it allows for superposition of entities, notwithstanding the assumption which I adopt that if any entities underlie a Unity, that Unity is *unique* (i.e. no other Unities are underlain by the same entities).

The theory of Unities is compatible with assuming that some wholes are not Unities - indeed, that by contrast with Unities, some wholes *are* fundamentally sums. In the version of the theory of Unities I put forward below, it is assumed that beside Unities there exist *collections* of Unities, where a collection of Unities is itself a Unity if, and only if, the collection has one member (i.e. itself). I will show that the mereological assumptions regarding such collections combine harmoniously with the very different mereological assumptions made with regard to Unities. It is particularly interesting that the assumptions made regarding collections are formally equivalent to classical mereology. The theory of Unities may be seen, therefore, as a multifaceted theory which includes classical mereology as a component applicable in certain contexts, though not in others.

Looking at the tree again, the theory will acknowledge, for example, the collection whose members are Root, Trunk, Branch1, and Branch2, as well as a collection whose members are Root and Trunk, or a collection whose members are Branch1 and Branch2. Similarly, there are collections of molecules. Any selected molecules, in fact, will be the members of a unique collection of molecules, a collection which is their mereological sum. Given that Root, Trunk, Branch1 and Branch2 underlie the tree, I say in this case that the collection of those elements *embodies* the tree. It is clear from what has been said, that I shall be assuming that distinct collections of elements may embody one and the same Unity.

With this introductory sketch of the suggestion in mind, I now turn to a more detailed and critical discussion of its principal components. The desire to show that the conception of wholes as Unities is compatible with ^awidely accepted core of mereological assumptions has made it necessary to explicate the formal structure of my assumptions. I beg the indulgence of the reader with the minuteness of the steps that sometimes needed to be traced. To arrive at an initial, more general impression of the theory of Unities the reader may skip the rest of this chapter, and turn now to Chapter 9, leaving out Subsection 9.2.1.

Section 8.2

Unities and Collections

8.2.1 *Collective and Distributive Whole-parts Relations and the Notion of a Unity*

Suppose a house has been built solely out of bricks, and that at present n bricks, $b_1 \dots b_n$, are parts of the house. Let us imagine that $n=10,000$. One can say, of course, that $b_1 \dots b_n$ are parts of the house. Furthermore, there is a common use of 'make up' (which I have described as part of the pre-theoretical framework of our discussion, in 3.1.4.3), according to which one can also say that $b_1 \dots b_n$ make up the house. According to this use of 'make up', however, it would be denied that $b_1 \dots b_{10}$, for example make up the house, although they are parts of the house. Similarly, it would be denied that b_3 and b_{42} make up the house, although they are parts of the house. Evidently 'the x s are parts of y ' does not mean the same as 'the x s make up y '.

According to this usage, use of the expressions 'are parts of' and 'make up' is closely bound up with the use of 'is a part of'. The following assumptions can be stated uncontroversially, I think, as constraining the semantic relations between these expressions.

Whereas regarding 'are parts of' there is the biconditional assumption:

- A1 for any x s, for all y ,
the x s are parts of y iff each of the x s is a part of y .¹

There is, regarding 'make up', merely the corresponding conditional assumption:

- A2 for any x s, for all y ,
if the x s make up y then each of the x s is a part of y .²

Furthermore, while it follows from A1 that³

¹ In other words: for any x s, for all y , the x s are parts of y iff for all z , if z is one of the x s then z is a part of y .

² In other words: for any x s, for all y , if the x s make up y , then for all z , if z is one of the x s then z is a part of y .

C1 for all xs , for all y , for all zs ,

if the xs are parts of y , and the zs are among the xs , then the zs are parts of y .

the example above involving bricks b_3 and b_{42} indicates that regarding 'make up', by contrast, one assumes:

A3 for some xs , for some y , for some zs ,

the xs make up y , and the zs are among the xs , but the zs do not make up y .

Similarly, it follows from A1 and the transitivity of *is a part of* that⁴

C2 for any xs , for all y , for all u ,

if the xs are parts of y , and y is a part of u , then the xs are parts of u .

By contrast, however, considering that bricks might make up a section of a house, without making up the house (of which that section is a part), it is assumed regarding 'make up' that:

A4 for some xs , for some y , for some u ,

the xs make up y , and y is a part of u , but the xs do not make up u .

Comparison of assumptions A1 with A2, C1 with A3, and C2 with A4, offers us a clear contrast between ways 'are parts of' and 'make up' are used (according to the familiar usage I am assuming). Taking these assumptions simply to explicate an existing linguistic usage, they may be conveniently referred to as 'the linguistic assumptions'.

³ Proof:

1. the xs are parts of y [assumption]
2. the zs are among the xs [assumption]
3. each of the xs is a part of y [from 1 and A1]
4. each of the zs is a part of y [from 2 and 3]
5. the zs are parts of y [from 4 and A1]
6. C1 [from 1, 2 and 5]

⁴ Proof:

1. the xs are parts of y [assumption]
2. y is a part of u [assumption]
3. each of the xs is a part of y [from 1 and A1]
4. each of the xs is a part of u [from 3 and 2]
5. the xs are parts of u [from 4 and A1]
6. C2 [from 1, 2 and 5].

In association with the contrast between the ways in which the expressions 'are parts of' and 'make up' are used, a distinction must be drawn between two notions which might be described as notions of a 'whole-parts relation'. If one claims that y bears to the x s the whole-parts relation, one might either mean simply that the x s are parts of y , or, more strongly, that the x s make up y . Since in some contexts it is the case that the x s are parts of y , though not that they make up y , it seems clear that two relations must be distinguished, relations which are ambiguously described by means of the expression 'the whole-parts relation'. In one sense of the latter expression, y bears the whole-parts relation to the x s if and only if the x s are parts of y . In the other sense, y bears the whole-parts relation to the x s if and only if the x s make up y . It is useful to have distinct expressions to designate these two relations. Let us define, therefore, the expressions 'is a whole with respect to' and 'is a whole which corresponds to' as follows:

D1 y is a whole with respect to the x s =_{def.}

the x s are parts of y

D2 y is a whole which corresponds to the x s =_{def.}

the x s make up y

With reference to the fact that the predicate in sentences of the form 'the x s are parts of y ' applies distributively, while the predicate in sentences of the form 'the x s make up y ' applies collectively,⁵ the whole-parts relations defined in D1 and D2 by means of these predicates, namely *is a whole with respect to* and *is a whole which corresponds to*, can be referred to respectively as 'the distributive whole-parts' relation and 'the collective whole-parts relation'. By contrast, the relation *is a part of* can be referred to as 'the part-whole relation'. Note that whereas the part-whole relation is dyadic, both the distributive and the collective whole-parts relations are neither dyadic nor even n -adic, for some n ; rather, they are multigrade relations.

It is noteworthy that the linguistic assumptions (A1-4, C1-2) guarantee (given definitions D1 and D2) that the distributive whole-parts relation can be

⁵ I say that a predicate of the form 'are F ', in sentences of the form 'the x s are F ', applies *distributively* if sentences of the latter type are equivalently analyzed as the conjunction ' x_1 is F and x_2 is F and ... and x_n is F ', where $x_1 \dots x_n$ are all and only the x s. A predicate of the form 'are F ' applies *collectively* if sentences of the form 'the x s are F ' cannot be analyzed in the way indicated. The distinction was explained above in Chapter 1, Section 1.3.

definitionally reduced to the part-whole relation, but do not similarly guarantee that such a definitional reduction is possible regarding the collective whole-parts relation. To see this, note that it follows from A1 and D1 that *y* is a whole with respect to the *x*s if and only if each of the *x*s is a part of *y*. That is, one might define 'is a whole with respect to' as follows:

D3 *y* is a whole with respect to the *x*s =_{def.}
each of the *x*s is a part of *y*.

This means that in formulating a mereological theory which accepted the linguistic assumptions and definitions D1-2, any assumptions adopted regarding the relation *is a part of* will fix at the same time the assumptions adopted regarding the relation *is a whole with respect to*.

By contrast, assumptions which one adopts regarding the relation *is a whole which corresponds to* are relatively independent of those adopted regarding the relation *is a part of*. For 'is a whole which corresponds to' cannot be defined in terms of 'is a part of', unless one makes further assumptions which would connect the relations designated by these expressions - beyond the assumptions discussed so far in the present Subsection.⁶

It is, indeed, very tempting to suppose that *is a whole which corresponds to* (no less than *is a whole with respect to*) should be definitionally reducible to *is a part of*, and so to seek further assumptions which would provide for a reductive definition. For it seems natural to suppose that *is a part of* and *is a whole which corresponds to* (as well as *is a whole with respect to*) are somehow correlative to one another, much like the relations *is smaller than or equal to* and *is larger than or equal to*. The sense that the former pair of relations is in some way analogous to the latter pair is perhaps strengthened by the observation that *is a proper part of* is plausibly assumed to be a partial ordering (i.e. a relation which is reflexive, antisymmetrical and transitive), just as much as *is smaller than or equal to* is.

⁶ The relation *are parts of, and weigh at least half as much as* complies (as can be seen by inspection) with the assumptions made regarding the relation *make up*, i.e. A2, A3 and A4. However, it is clear that the familiar use of 'make up' which I am discussing here is distinct from that of 'are parts of, and weigh at least half as much as'. Thus assumptions A2, A3 and A4 do not suffice to define 'make up' in terms of 'is a part of'. In particular, given the definition of 'is a whole which corresponds to' in terms of 'make up' (D2), they do not suffice to define 'is a whole which corresponds to' in terms of 'is a part of'.

Whether for this reason, or perhaps due to historical connections between mereology and class theory,⁷ it seems to be very widely presupposed in current discussions concerning wholes and parts that *is a whole which corresponds to* is adequately accounted for as the relation *is a sum of*:

D4 y is a whole which corresponds to the $xs =_{\text{def.}}$

y is a sum of the xs

Given the conventional definition of 'is a sum of' in terms of 'is a part of', this is to presuppose the following definition of 'is a whole which corresponds to' in terms of 'is a part of' (recall that 'overlaps' is defined in terms of 'is a part of': for u overlaps v iff for some w , w is a part of u and w is a part of v):

D4' y is a whole which corresponds to the $xs =_{\text{def.}}$

for all z , z overlaps y iff z overlaps one of the xs .

Assumption D4 (or D4') is an implicit characteristic of what I have described as 'the conception of wholes as sums'. It appears to be so deeply ingrained that rarely if ever does the possibility surface in contemporary debates, that this assumption

⁷ If one is guided, in developing a theory of wholes and parts, by the theory of classes, one is likely to assume that *make up* is reducible to *is a part of*, and thus that *is a whole which corresponds to* is reducible to *is a part of*. For one might draw a distinction regarding classes which is parallel to the distinction between 'are parts of' and 'make up' discussed above regarding wholes. That is, one might distinguish between 'are members of', on the one hand, and 'form', on the other hand, so that if $s_1 \dots s_n$ are all the stamps in my stamp-collection, and if C is the class of those stamps, the following claims can be affirmed:

1. $s_1 \dots s_n$ are members of C
2. $s_1 \dots s_n$ form C
- and
3. s_2, s_{17} , and s_{225} are members of C
- but
4. s_2, s_{17} , and s_{225} do *not* form C

The relation *form*, however, as holding between members (taken together) and classes, is straightforwardly reducible to the relation *is a member of*, given the assumption of extensionality of classes:

the xs form $y =_{\text{def.}}$

for all z , z is one of the xs iff z is a member of y

If the theory of wholes and parts is conceived of as closely analogous to the theory of classes (recall that Lesniewski took his mereology to be a theory of what he called 'collective classes'), then one should expect that *make up* (which corresponds to *form* in the theory of classes) be reducible to *is a part of* (which corresponds to *is a member of* in the theory of classes).

might be rejected. I have noted, however, that no reduction of *is a whole which corresponds to* to the relation *is a part of* is entailed by the linguistic assumptions outlined above. Furthermore, the analogy between the pair *is a part of* and *is a whole which corresponds to* and the pair *is smaller than or equal to* and *is larger than or equal to* (an analogy which is likely to suggest that a reduction of *is a whole which corresponds to* should be available in principle) is far from perfect. In particular, whereas both *is smaller than or equal to* and *is larger than or equal to* are binary relations, *is a part of* - as noted above - is binary but *is a whole which corresponds to* is not (indeed, nor is it *n*-ary, for some *n*; rather, it is a multigrade relation).⁸ And the former pair presents no parallel to the distinction between a distributive and a collective whole-parts relation.

It has been shown in Part II that the conception of wholes as sums involves great difficulties. It would therefore seem to be highly desirable to explore an alternative conception, if such were available. The discussion of the present Subsection indicates a direction in which such an alternative might be sought. A mereological theory might be considered, which rejects definition D4 (as well as D4'). In particular, this involves the proposal that a whole which corresponds to some entities need not be a sum of those entities. Wholes of this sort I shall describe as 'Unities'.

Use of 'Unity' to designate a kind of comprising entity, or a kind of whole, is not unprecedented. For example, Russell uses the term to describe one of the three notions of a whole which he distinguishes in *The Principles of Mathematics*.⁹ My use of the term, however, is in no way meant to signify any connection with such former uses.

It is important not to confuse the term 'Unity' with the term 'unity' (lower case 'u') which I made frequent use of in Part I. The latter was, and will continue to be, used to designate the property *is unified*.

⁸ If we describe the converse of the part-whole relation *is a part of* as the whole-part relation (note 'part' in the singular), then this pair of relations, the part-whole relation and the whole-part relation - both being binary relations - is more closely analogous to the pair *is smaller than or equal to* and *is larger than or equal to*.

This analogy, however, only gives grounds for supposing that the whole-part relation is reducible to the part-whole relation, and not for supposing that the collective whole-parts relation is similarly reducible.

⁹ See Russell 1903, 139f.

The term 'Unity' is chosen principally because of what seems to be its intuitive suitability, as well as the fact that other candidates either strongly connote notions which need to be dissociated from the notion of a Unity (e.g. 'unit', 'monad', 'singleton', 'integral whole'), or are simply too artificial sounding and awkward (e.g. 'unit-entity', 'uniton'). Furthermore, 'Unity' seems to me particularly appropriate, since I take the notion to offer a plausible interpretation for what Plato describes in *Theaetetus* as a "single form produced out of [the letters], having its own single nature - something different from the letters."¹⁰

The theory of Unities will require other specialized terms. Proper parts of Unities are called 'pre-elements'. *Immediate* proper parts of Unities are called elements.¹¹ A relation designated by 'underlie' plays a role in the theory of Unities which is analogous to the role played by 'compose' in classical and neoclassical mereology. If each of the *xs* is an element of *y*, and *y* is a whole which corresponds to the *xs*, then the *xs* underlie *y*. All this will become much clearer in the course of the systematic presentation of the theory in 8.3.

A striking difference between the conception of wholes as Unities and the conception of wholes as sums appears when the following assumption is introduced, which it is possible to adopt consistently only because D4 has been rejected.

- A5 for some Unity *y*, for some *xs*, for some *zs*,
y is a whole which corresponds to the *xs*, and
y is a whole which corresponds to the *zs*, and yet
the *zs* are properly among the *xs* (i.e. the *zs* are some *but not all* the *xs*)

Thus if the house, for example, is a Unity, then it is consistent with A5 that the house is a whole which corresponds to the 10,000 bricks, and yet at the same time a whole which corresponds to 9,999 of the bricks. To assume this would be clearly

¹⁰ Plato *Theaetetus*, 203e3-5. The Greek text reads as follows:

... ek ekeinOn hen ti gegonos eidos, idean mian auto hautou ekhon, heteron de ton stoikheion.

Regarding the connection to Plato's discussion in *Theaetetus*, see Introduction to Thesis.

¹¹ 'immediate proper part' is defined as follows:

x is an immediate proper part of *y* =_{def.}

x is a proper part of *y* and

there is no *z* such that *x* is a proper part of *z* and *z* is a proper part of *y*.

See Simons 1987, 108, as well as definition D7 Subsection 8.3.2.

inconsistent with the conception of wholes as sums, given its adherence to D4 (and D4'). Nothing can be at the same time a sum of the 10,000 bricks and a sum of the 9,999 bricks. For the 10,000th brick overlaps any sum of the 10,000 bricks, and no sum of the 9,999 bricks.¹²

To take another example, suppose Head, Neck, Torso, Leg1, Leg2, Leg3, Leg4 and Tail are parts of a certain cat at some moment of time *t*. Suppose Tibbles is a whole which corresponds to all those parts, whereas Tib is a whole which corresponds to all those parts *except Tail*. If one accepts D4 then one is bound to take Tibbles and Tib to be distinct from one another. If, however, denying D4, one affirms A5, it is open to one to claim that Tibbles is identical to Tib. Section 9.3 is devoted to discussing a familiar paradox often presented in terms of this example. I argue there that an approach based on the theory of Unities offers a better account of the paradox than do approaches based on the conception of wholes as sums.

Even if one has denied D4 (and D4'), and accepted A5, it is tempting to seek for alternative ways (replacing D4') in which *is a whole which corresponds to* might be reduced to *is a part of*. Even if one accepts the plausibility of affirming that the house, for example, is a whole which corresponds to some 9,999 of its 10,000 bricks, one would presumably deny that the house is a whole which corresponds, say, to some 20 of its bricks. One would assume that it should be possible to demarcate, at least roughly, the boundary between cases where the *xs* (each being a part of *y*) are such that *y* is a whole which corresponds to them, and cases where they are such that *y* is not a whole which corresponds to them.

One might propose, for example, that some *xs* are such that the house is a whole which corresponds to them, if and only if each of the *xs* occupy (collectively) roughly 95% of the region of space occupied by the house. Perhaps other conditions might be added, for example regarding relative proximity of the *xs*.

¹² To verify this claim, suppose

c is a sum of $b_1 \dots b_{n-1}$

d is a sum of $b_1 \dots b_{n-1}, b_n$

From the definition of 'sum' it follows that

for all *x*, *x* overlaps *c* iff *x* overlaps one of $b_1 \dots b_{n-1}$

for all *x*, *x* overlaps *d* iff *x* overlaps one of $b_1 \dots b_{n-1}, b_n$

This entails (assuming b_n overlaps none of the other *bs*) that b_n does not overlap *c*, but that it does overlap *d*.

However, even if one managed to justify the claim that the house is a whole which corresponds to the *xs* if and only if such conditions are fulfilled, it seems unlikely that the same conditions could be justified regarding entities of other types, that is, regarding horses, trees, or stones, instead of houses. Perhaps for stones, for example, one should replace 95% with 60% (in the condition noted in the former paragraph). Perhaps for horses, and for trees, one should develop conditions which are sensitive to differences between parts as regards their relative importance for the existence of the whole.

Such reflections make it plausible to assume that, having set aside the conception of wholes as sums, no general conditions can be formulated to replace D4', independently of the *sort* of entities concerned. I shall take it as a working assumption, therefore, that no sortally-independent condition is available which would replace D4', and hope to show that this is not a drawback but rather an advantage of the theory of Unities.

With regard to its rejection of D4', and of any alternative (sortally independent) definition of 'is a whole which corresponds to' in terms of 'is a part of', the theory of Unities is appropriately described as being *holistic*, in a sense in which no theory which embraces the conception of wholes as sums can be. For whereas a theory of the latter type takes the notion of a whole to be conceptually derivable from the notion of a part (for both 'is a whole with respect to' and 'is a whole which corresponds to' are definable in terms of 'is a part of'), the theory of Unities takes the notion of a whole not to be so derivable (for although 'is a whole with respect to' is definable in terms of 'is a part of', 'is a whole which corresponds to' is not).

8.2.2 *Wholes and Sums*

To reject the conception of wholes as sums, is not the same as rejecting the claim that all wholes are sums. By rejecting D4, one is rejecting the claim that a whole which corresponds to the *xs* is necessarily a sum of the *xs*. One is not rejecting the claim that a whole which corresponds to the *xs* is necessarily a sum of *some zs*. To be clearer on this point, let us distinguish between saying that an entity

is *fundamentally* a sum, and saying that it is *peripherally* a sum. An entity is peripherally a sum, if, simply, there are some *xs* of which it is a sum:

D5 *y* is peripherally a sum =_{def.}
for some *xs*, *y* is a sum of the *xs*.

By contrast, an entity is fundamentally a sum if in addition to being peripherally a sum, it is such as to be a sum of any *xs* to which it is a corresponding whole:

D6 *y* is fundamentally a sum =_{def.}
1. for some *xs*, *y* is a sum of the *xs*
2. for any *xs*, if *y* is a whole which corresponds to the *xs*, then *y* is a sum of the *xs*.

It is entailed by the conception of wholes as sums, given its acceptance of D4, that all wholes¹³ are fundamentally sums. Indeed, it is because this conception is committed to the claim wholes are fundamentally sums, rather than the weaker claim that wholes are peripherally sums, that it is particularly appropriate to describe this conception as the conception of wholes as sums.

The theory of Unities, in as much as it accepts assumption A5, must reject the claim that all wholes are fundamentally sums, this claim being clearly inconsistent with A5. It is consistent with A5, however, to accept the claim that all wholes are peripherally sums. Moreover, if one assumes that *is a part of* is transitive, one is bound to accept that if the *xs* are *all* the parts of *y* (using 'all the parts' here in the strict sense, rather than the usual qualified sense, explained in 3.1.4.4) then *y* is a sum of the *xs*. Both because I shall be accepting the transitivity of *is a part of* in what follows, and because I take it to be intrinsically reasonable to claim that a whole must be a sum of all its parts (again, taking 'all its parts' in the strict sense), I shall assume that all wholes are peripherally sums, whether or not they are Unities:

A6 for all *x*, if *x* is a whole then *x* is peripherally a sum.

¹³ Use of expressions of the form '*y* is a whole' was explained in Item 3.1.4.1.

8.2.3 *Classical, Neoclassical, and Unity-theoretical Assumptions*

It is useful to summarize the foregoing introduction to the notion of a Unity by comparing constraints on relations between wholes and sums, as assumed in classical mereology, in neoclassical mereology, and in the theory of Unities.

There are three noteworthy features of wholes as conceived of in classical mereology: universal existence, uniqueness, and coextensivity of parts. These features have been discussed in Part II, and may be explained now using the terminology introduced in the current Subsection as follows:

universal existence:

wholes are universally existent iff for any xs , there is a whole which corresponds to those xs .

uniqueness:

wholes are unique iff for any xs , for all y , for all z , if y is a whole which corresponds to the xs and z is a whole which corresponds to the xs , then y is identical to z .

coextensive determination:

wholes are coextensively determined iff for all u , for all v , for any xs , for any ys , if u is a whole which corresponds to the xs , and v is a whole which corresponds to the ys , then u is identical to v only if the xs are coextensive with the ys .¹⁴

Neoclassical mereology rejects either universal existence, or uniqueness, or both. However, both classical and neoclassical mereology are compelled to maintain coextensivity of parts, since this follows from the way the term 'sum' is defined (as explained in 7.1), and from acceptance of definition D4, whereby wholes are made out to be fundamentally sums.

The theory of Unities differs from both classical and neoclassical mereology in rejecting coextensivity of parts. It is able to do so by rejecting the claim embodied in D4, according to which wholes are fundamentally sums. As usual with neoclassical

¹⁴ The xs are coextensive with the ys iff for all z , z overlaps one of the xs iff z overlaps one of the ys . See Sections 2.3 and 7.1 above.

mereology, and in contrast with classical mereology, the theory of Unities rejects universal existence of wholes. As for uniqueness, its position might be roughly described as midway between classical and neoclassical mereology. A precise explanation of this issue, however, is best left to the systematic presentation in the next Section.

8.2.4 *Unities and Collections*

Although it is of much interest to investigate possible applications of the theory of Unities *outside* the context of the study of concrete entities,¹⁵ I shall for the purposes of the present work confine myself to concrete entities. It will be useful, at this stage, to state briefly the chief intended interpretation of the central term of the theory, namely 'Unity', to facilitate a less abstract discussion: I take the term 'Unity' to apply to concrete *individuals*. Furthermore, I propose the conception of such individuals as Unities as a comprehensive alternative to their prevalent conception as a neoclassical mereological sum.

It is noteworthy, however, that given this interpretation, the theory of Unities is compatible with the view that not all among concrete entities are individuals. In particular, it will be shown in detail in the next Section that the theory of Unities is compatible with the view that for any Unities, there exists a *collection* of those Unities. Such a collection is itself a Unity if, and only if, it has only one member. Since a multi-membered collection of Unities is not itself a Unity, and individuals are assumed to be Unities, it is compatible with the theory that entities - multi-membered collections - exist, which are not individuals.

The notion of a collection, in this context, is understood in accordance with the account offered in Part I. It was there suggested that the notion of a collection is indeterminate in various respects (for example, in respect of the requirement that a collection should be unified, or that it be unique, or that the relation *is a part of*, as applying to collections, be transitive). It transpired in Part II that the notion of a

¹⁵ A comment to this effect was mentioned in the Introduction to this work.

classical mereological sum is one plausible determination of the notion of a collection. It is this determined notion of a collection that I adopt in the present context, in supplementing the theory of Unities with a theory of collections.

Therefore, the theory of collections presented here will have formal features which precisely parallel the formal features of classical mereology. Thus if a_1, a_2, \dots, a_6 are Unities, and A is a collection of these Unities, A is naturally described as a classical mereological sum of a_1, a_2, \dots, a_6 , with only one qualification: A is not itself an individual, for it is a multi-membered collection. Traditionally, classical mereology is assumed to be a calculus of *individuals*, such that the classical sum of any individuals is itself an individual. Although the theory of collections presented here is formally indistinguishable from classical mereology, it cannot be described as a calculus of *individuals*, but rather as a calculus of *collections* of individuals (i.e. collections of Unities).

This difference, however, does not arise from any formal difference between the theories, but rather from a difference in the way the terms of the theories are respectively interpreted. On the traditional interpretation of classical mereology as a calculus of individuals, the variables of the theory range over individuals; on the present interpretation of classical mereology as a calculus of collections, the variables of the theory range over collections of Unities.¹⁶

To supplement the theory of Unities with such a theory of collections is not to concede that the theory of Unities presupposes classical mereology. The assumptions required for a theory of Unities are presented in the next Section separately from the supplementary assumptions required for a complementary theory of collections, and thus it will be clear that the latter theory is not presupposed by a theory of Unities. In particular, classical mereology is not presupposed.

However, given that classical mereology is an internally consistent theory,¹⁷ and given that on the interpretation adopted here one is not committed to the existence of arbitrary sum-*individuals*, nor even, as we shall see, to the view that

¹⁶ Note, for comparison, that Lewis (1991) requires an interpretation of mereology according to which the values of its variables are *classes*. That is, they are not only not *individuals*, but are not even concrete entities. Collections of Unities, on my account, are not individuals, but they are concrete entities.

¹⁷ See Simons 1987, 110, footnote 23.

distinct individuals cannot be at the same place at the same time, there seems to be little harm in accepting classical mereology thus interpreted.

On the other hand, accepting the existence of collections, and in particular collections understood as classical sums, is in my view both inherently plausible and convenient. The existence of such collections would provide a straightforward explanation of, and justification for, our talk of such entities as a collection of pebbles or a group of dots on the blackboard. Furthermore, accepting the existence of such collections in the present context allows us to demonstrate how a well established traditional theory such as classical mereology can be smoothly combined with assumptions about Unities.

To obtain a clearer first impression of the ontological picture that results, consider again our example of the house h and its n bricks, the bs . In order to have concrete numbers in mind, suppose $n=100$. Suppose each of the bricks is a Unity, and the house itself is a Unity. The assumptions so far do not entail that there are more than 101 Unities. Each of these Unities is a single-membered collection. However, the assumptions do entail that there is an unimaginably larger number of (non-single-membered) collections. For any two bricks, there is a collection of those bricks; and similarly for any three bricks, four bricks, ... , n bricks. To be precise, the number of collections of bricks works out to be $2^{100} - 1$. That is to say, there are roughly 1,267 billion billion billion distinct collections of bricks.

Given the result of our discussion in Section 7.2, however, any of these numerous collections is collectively identical to the bricks that compose it. This means that the assumption that such collections exist can be said in the strictest sense to involve no extension of our ontological commitments. Granted the existence of the 101 Unities, the assumption that arbitrary sums of those Unities exist introduces no new entities, that is, no entities that are not collectively identical to some or all of the 101 existing Unities.

A collection, being an entity which complies with classical mereological assumptions, can be described (in terms explained in the former Section) as being *fundamentally* a sum. A Unity, by contrast, is only *peripherally* a sum. Now suppose the as are some, but not all of the bricks (i.e. suppose the as are properly among the bs), and suppose A is the collection of the as and B is the collection of the bs . It follows that A is a whole which corresponds to the as but not to the bs , whereas B is

a whole which corresponds to the *bs* but not to the *as*. The house, however, since it is assumed to be a Unity, may be (though of course need not be) at the same time a whole which corresponds to the *as* and to the *bs*.

Let us call the collection of the *bs* the *complete* collection of bricks (at time *t*). It is one of the noteworthy features of the theory of Unities that it allows us to draw a clear distinction, in a case such as this, between the house and the complete collection of bricks that underlie it at *t*, irrespective of any assumptions regarding the disparity of conditions under which the house and the collection might survive, respectively.

The distinction between Unities and multi-membered collections is a distinction between two fundamentally different notions of a whole. These notions need not be viewed as competing with one another, but rather as complementing one another. As will emerge more clearly from the systematic presentation of assumptions in the next Section, a collection is a whole which is identical to its parts. A Unity, on the other hand, is a whole which is not identical to its parts.

Both Unities and multi-membered collections are *single* entities. It was argued in Part I that singleness does not conflict with identity to many parts. However, in that a Unity is not identical to its parts, nor, indeed, to any *xs*, if the *xs* are greater than 1 in number, Unities are *monadic* entities.¹⁸ Multi-membered collections, by contrast, are *non-monadic* entities. Entities of both types have parts. But the character of the respective relations between these entities and their parts is fundamentally different. On my analysis, the main difference lies in the difference between the species of the relation *is a whole which corresponds to* which apply respectively to collections and Unities, and in the attendant difference between entities that are fundamentally sums and entities that are only peripherally sums.

It is true that according to the account I have offered, neoclassical sums, too, are wholes conceived of as non-identical to their parts (see relevant comments in Section 7.2). However, we have seen in Chapter 6 that the notion of a neoclassical sum is deeply problematic, and if it has to be rejected, it is important to realize that we have an alternative conception of a whole, one which also requires the conception of a whole as non-identical to its parts. Moreover, the notion of a Unity

¹⁸ See Section 1.2 above - in particular 1.2.6 and 1.2.7.

goes much further than the notion of a neoclassical sum in distinguishing itself from the contrasting notion of a whole which is identical to its parts. The grounds offered by neoclassical mereology for distinguishing between a whole and its parts depend on extra mereological considerations, such as the survival of individuals under circumstances of flux of parts and of changes in the relations between parts. The theory of Unities provides us in addition with intrinsic mereological grounds for distinguishing between a whole and its part⁵(if the whole is a Unity).

Furthermore, neoclassical mereology seems unable to offer a conception of a whole which is (in terms of the discussion of Subsection 6.2.2) presently discernible from a superposed classical sum. By contrast, it will be seen that a Unity *is* presently discernible from a superposed classical sum. Thus, although neoclassical mereology proposes wholes which are not identical to their parts, these are wholes which are at all times (presently) indiscernible from classical sums, and thus they are at all times (presently) indiscernible from all the parts (taken collectively). This is not a compelling notion of a whole which is not identical to its parts. The contrast between wholes which are identical to their parts and wholes that are not, which the theory of Unities offers us, I suggest, is much starker.

Section 8.3

Principles for a Theory of Unities

I now turn to show how basic principles may be constructed for a theory of wholes and parts in which the notions of a Unity and of the relation *underlie* play a central role. What I offer here cannot, by any means, be viewed as a fully developed theory. It will be evident that a theory based on the core notion of a Unity may be developed in many and varied ways. My aim is not to survey all the possibilities that arise, but rather to present an initial picture which can serve as an indication of the way a more complete and well rounded theory could be developed. In systematic terms, the principles I have collected are rather loosely organised. I have attempted to put together a picture which makes what I take to be important features of the theory clear, rather than one which emphasizes the inferential relations governing the system as a whole. In particular, I make no claim that the version I present cannot be shown to be founded on a smaller number of principles, from which some of the others could be derived.

As before, I leave the temporal modification of predicates such as 'is a part of', 'is coextensive with', 'is a sum of', 'underlie', etc. almost completely implicit, to simplify the presentation. Supplementing the account with explicit temporal modification of these predicates presents no problems of principle.

8.3.1 *Preliminary Assumptions Regarding Unities and Collections*

We begin by assuming that some entities are Unities, viewing them as belonging to a corresponding ontological category, the category of Unities. That is,

A1 for some x , x is a Unity.

If x is a Unity, then x is a single entity, in the sense explained in Part I. In that discussion it was urged that the claim that x is a single entity is compatible with the claim that x is (collectively) identical to many entities. However, we shall assume

that a Unity cannot be identical to many Unities (nor to many entities of other types). To describe this feature of Unities, we may use the term 'monadic', defining it as follows:

D1 y is monadic =def.

for any xs , y is identical to the xs only if the xs are 1 in number

With regard to Unities, therefore, we assume:

A2 for all x , if x is a Unity then x is monadic.

It would seem that the only context in which it might be plausible to suggest that a single entity is identical to many distinct entities is a case in which the former is a whole which corresponds to the latter.¹⁹ In order to allow, as we shall be assuming, that a Unity u might both be a whole which corresponds to the xs and a whole which corresponds to the ys , even though the xs are not coextensive with the ys , we would seem to be compelled, at least with regard to those cases, to assume that u is neither identical to the xs nor to the ys .

There are other reasons for assuming A2, some deriving from the mereological assumptions that are presented below, others from considerations associated with the intended interpretation of Unities as individuals.²⁰ Examining the details of these considerations would lead us far astray, however, and even if restricting the scope of A2²¹ is compatible with other requirements of a theory of Unities, the discussion is simplified by adopting A2 in its present unrestricted form.

Next, we introduce collections of Unities. For some entities, the xs , we assume that there exists a unique entity which is a collection of the xs . Thus we assume:

A3 for some u , and for some xs , u is a collection of the xs .

¹⁹ The claim that a whole is identical to all its parts is not without its sympathizers, as we have seen in Chapter 6. I am not aware, however, that the claim that a single entity is identical to many entities (taken collectively) has ever been put forward with regard to concrete entities in connection with cases in which the many entities are not taken to be *parts* of the relevant single entity (though perhaps such a claim is made with regard to universals by some of the theorists who conceive of universals as repeatables - see Armstrong 1989; Bigelow 1988).

²⁰ Similar considerations lead to the conclusion that a neoclassical conditioned sum, for instance, is not identical to its parts. An example of such a consideration is the observation that in some cases the parts of an individual survive the individual's destruction.

²¹ E.g. assuming A2 only in cases where x has more than two elements.

- A4 for all u , for all v , and for any xs , if u is a collection of the xs and v is a collection of the xs , then $u=v$.

Collections we take to be always collections of Unities. And such collections exist universally. That is,

- A5 for all u , and for any xs , if u is a collection of the xs then each of the xs is a Unity.
- A6 for any xs , if each of the xs is a Unity, then for some y , y is a collection of the xs .

Suppose three bricks, b_1 , b_2 , and b_3 , taken collectively, are a value of the variable ' xs ' (note that because ' xs ' is a plural variable, it may take as *one* of its values more than one entity). Then on our assumptions, if there is a collection of the xs when the variable takes b_1 , b_2 , and b_3 (collectively) as its value, then each of the bricks is a Unity. Conversely, if each of the bricks is a Unity, then there exists a collection of the xs , when ' xs ' takes b_1 , b_2 , and b_3 (collectively) as its value.

Similarly (noting that although a plural variable may take more than one entity as its value, it may also take only one entity as its value), if there is a collection of the xs when ' xs ' takes only one of the bricks, say b_1 , as its value, then that brick is a Unity. And coversely, if b_1 is a Unity, then there exists a unique collection of the xs , when ' xs ' takes b_1 as its value.

The notion of a member is defined as follows:

- D2 y is a member of u =_{def.}
for any xs , if u is a collection of the xs then y is one of the xs

It follows that²²

²² That C1 is a consequence of our assumptions is shown by the following argument:

1. y is a member of u =_{def.}
for any xs , if u is a collection of the xs then y is one of the xs [D2]
2. for any ys , for all u , for any xs ,
all of the ys are members of u (i.e. each of the ys is a member of u) iff
if u is a collection of the xs then each of the ys is one of the xs . [from 1]
3. for any ys , for all u , for any xs ,
all of the ys are members of u iff
if u is a collection of the xs then the ys are among the xs . [rewriting 2]

- C1 for any ys , for all u , for any xs ,
 all and only the ys are members of u iff
 if u is a collection of the xs then
 the $ys =$ the xs .

The notions of a sub-collection and a proper sub-collection are defined as follows:

- D3 u is a sub-collection of $v =_{\text{def}}$
 for all x (x is a member of u only if x is a member of v).
- D4 u is a proper sub-collection of $v =_{\text{def}}$
 u is a sub-collection of v and for some x , x is a member of v
 but not of u .

We assume that a single-membered collection is identical to its member:

- A7 for all u and for all x , if u is a collection whose only member is x ,
 then $u=x$.

-
4. for any ys , for all u , for all z ,
 only the ys are members of u iff
 if z is a member of u then z is one of the ys .
 [analysis of 'only']
5. for all z , for all u , for any ys , for any xs ,
 if z is a member of u then z is one of the ys
 iff
 if u is a collection of the xs then
 if z is one of the xs then z is one of the ys .
 [from D2]
6. for all z , for all u , for any ys , for any xs ,
 if z is a member of u then z is one of the ys
 iff
 if u is a collection of the xs then
 the xs are among the ys .
 [from 5]
7. for any ys , for all u , for any xs ,
 only the ys are members of u iff
 if u is a collection of the xs then the xs are
 among the ys .
 [from 4 and 6]
8. for any ys , for all u , for any xs ,
 all and only the ys are members of u iff
 if u is a collection of the xs then
 the $ys =$ the xs .
 [from 3 and 7]

Assumption A7 guarantees that a single-membered collection is a Unity:²³

C2 for all u , if u is a single-membered collection then u is a Unity.

From A6 and A7, we may infer:²⁴

C3 for all u , u is a Unity if and only if it is a single-membered collection.

However, this still does not preclude a many-membered collection from being a Unity, for our assumptions so far are consistent with a single-membered collection being identical to a many-membered one. We rule out this possibility, and simplify the relations between Unities and collections, by assuming as follows:

A8 for all u , if u is a collection with more than one member, then u is not a Unity.

Assuming that there are at least two Unities, then C3, A6, and A8 guarantee that the ontological category of Unities is strictly a sub-category of the category of collections.

²³ Proof:

1. for all u , if u is a collection of which all and only the x s are members, then u is a collection of the x s. [from C1]
2. for all u , if u is a collection of which all and only the x s are members, then each of the x s is a Unity [from 1 and A5].
3. if u is a collection whose only member is x , then x is a Unity. [from 2]
4. Therefore, if u is a collection whose only member is x , then u is a Unity. [from 3 and A7]

²⁴ Proof:

1. for all u , if u is a single-membered collection, then u is a Unity [from C2]
2. for all u , if u is a Unity then for some y , y is a collection of the x s, and the x s = u . [from A6]
3. for all y , for any x s, if y is a collection of the x s then y is a collection of which all and only the x s are members. [from C1]
4. for all u , if u is a Unity then for some y , y is a collection whose only member is u . [from 2 and 3]
5. for all u , if u is a Unity then for some y , y is a collection whose only member is u , and $y = u$. [from 4 and A7]
6. for all u , if u is a Unity then u is a single-membered collection. [from 5]
7. for all u , u is a Unity iff it is a single-membered collection [from 1 and 6]

8.3.2 *Mereological Assumptions Regarding Unities and Collections*

We lay down as a foundation for a mereological theory involving the notion of a Unity definitions and assumptions which demonstrate the wide common ground between this theory, on the one hand, and classical and neoclassical mereology on the other.

Treating 'proper part' as a primitive term, we define 'improper part', 'part', 'immediate part', 'overlap', 'disjoint' and 'sum' in agreement with common mereological usage, adding to the list the term 'coextensive', of which we have already made use in the discussion above. Note that although the theory of Unities is compatible with the phenomenon of being in the same place at the same time, we do not require to resort to the rather extreme measure which Simons requires for this purpose, that of denying D5, an assumption maintained both in the classical mereological view and, it would seem, by common sense.²⁵

- D5 x is a part of $y =_{\text{def.}}$
 x is a proper part of y or x is identical to y
- D6 x is an improper part of $y =_{\text{def.}}$
 x is a part of y and x is not a proper part of y
- D7 x is an immediate proper part of $y =_{\text{def.}}$
 x is a proper part of y and it is not the case that
for some z , x is a proper part of z and z is a proper part of y .
- D8 x overlaps $y =_{\text{def.}}$
for some z , z is a part of x and z is a part of y
- D9 x is disjoint from $y =_{\text{def.}}$
it is not the case that x overlaps y
- D10 y is a sum of the x s $=_{\text{def.}}$
for all z , z overlaps y iff z overlaps at least one of the x s

²⁵ See Simons's discussion of issues involved here at Simons 1987, 112f.

D11 the x s are coextensive with the y s =_{def.}

for all z , z overlaps at least one of the x s iff z overlaps (at least) one of the y s

Next, we lay down widely accepted assumptions governing the relation *is a proper part of*, expressing the asymmetry and transitivity of the relation,²⁶ as well as its compliance with the Weak Supplementation principle:²⁷

A9 for all x , for all y , if x is a proper part of y then y is not a proper part of x .

A10 for all x , for all y , for all z , if x is a proper part of y and y is a proper part of z then x is a proper part of z .

A11 for all x , for all y , if x is a proper part of y then for some z , z is a proper part of y and z is disjoint from x .

²⁶ See Simons 1987, 26-28; 362. I have commented on several opportunities above on the controversiality of the assumption of transitivity of *is a proper part of* (A8). The theory of Unities does not depend on making this assumption. However, since it is compatible with this assumption, and since the assumption is characteristic of both classical and neoclassical mereology, it helps to demonstrate the common grounds between the latter theories and the theory of Unities if the assumption is kept.

²⁷ The Weak Supplementation principle places a somewhat unnatural constraint on the theory of Unities, in that it implies that no Unity can exist which has one and only one element at some time t . Simons comments that Brentano denied this principle (and that a somewhat similar view to Brentano's is held by Kit Fine). However, he asks,

How could an individual have a *single* proper part? That goes against what we mean by 'part'. An individual which has a proper part needs other parts in addition to *supplement* this one to obtain the whole. (Simons 1987, 26).

The appeal to what we mean by 'part', however, would seem to go no less against the view that Simons *does* maintain, that an improper part of a whole need not be identical to its whole (see Simons 1987, 112; on the basis of this view Simons denies the *Strong* Supplementation principle). Thus it is not clear that a theory which upholds Weak Supplementation and denies Strong Supplementation is more faithful to "what we mean by 'part'" than a theory which upholds Strong Supplementation and denies Weak Supplementation (a theory of Unities of the latter type would be more flexible than one of the former type).

Countering Simons's point in the quoted passage, it is arguable that the intuition that a whole must be in some way "more" than anything that is its proper part is satisfied by the observation that the whole simply has more parts than any of its proper part^s does. If y has a proper part x , we need not assume that y has an additional proper part z which is disjoint from x , to guarantee that y has more proper parts than x does. After all, transitivity alone guarantees that y has more proper parts than x does - y has all the proper parts which x does, and in addition it has x itself as a proper part (while x , of course, does not have itself as a proper part). However, again applying a consideration similar to the one noted in the preceding footnote, I have chosen to maintain the Weak Supplementation principle in the present context.

In addition to assumptions regarding the relation *is a proper part of*, the discussion of Subsection 8.2.1 has indicated the importance of making explicit our assumptions about the relation *is a whole which corresponds to*. In classical and neoclassical mereology this relation is taken to be accounted for as the relation *is a sum of*, and since the latter relation is analyzable in terms of the relation *is a proper part of* (see definitions D5, D8 and D10 above), the need does not arise to treat 'whole' as an additional primitive term of the system, in addition to 'proper part'. However, since^{as} against the view implicit in classical and neoclassical theories we reject the account of wholes as sums, we are obliged, in effect, to take 'whole' as primitive, no less than 'proper part'.

We accept, of course, that there are analytic connections between 'whole' and 'proper part'. Assumption A12 below, for example, is an expression of such connections. However, we deny that these analytic connections between the terms suffice to establish a relation of interdefinability between them. And therefore the assumptions made above regarding the relation *is a proper part of* do not determine the assumptions adopted regarding the relation *is a whole which corresponds to*. We therefore introduce the following assumptions, which are motivated by our discussion of Subsection 8.2.1.

A12 for all y , for any xs , if y is a whole which corresponds to the xs
then each of the xs is a part of y .

A13 it is not the case that for all y , for any xs , for any zs ,
if y is a whole which corresponds to the xs then if the zs are among the xs
then y is a whole which corresponds to the xs .

A14 for all y , for any xs , if y is a whole which corresponds to the xs
then for some zs , the xs are among the zs and y is sum of the zs .

Definitions D5-D11 and assumptions A9-A14 can be taken to specify basic features of the relations *is a proper part of* and *is a whole which corresponds to*. This does not mean, however, that we take there to be only one type of relation which may legitimately be described by 'is a proper part of' (nor only one which may be described by 'is a whole which corresponds to'). We may take it that these relations are related to others as generic to specific. That is to say, there is the generic *is a*

proper part of (and *is a whole which corresponds to*) which is found in various species. The species of *is a proper part of* share with the genus the formal features outlined above. There may, however, be formal differences between different species of the generic relation, differences which are compatible with the shared core of features.

We shall assume, in particular, that there are two distinct species of *is a proper part of*, and correspondingly of *is a whole which corresponds to*. One species of *is a proper part of* is called 'is a pre-element of', and is a relation which holds between x and y only if both x and y are Unities. The corresponding species of *is a whole which corresponds to* is called 'is underlain by'. A second species of *is a proper part of* is called 'is a proper sub-collection of', and is a relation which holds between x and y only if y is not a Unity (recall that we are assuming that all concrete entities fall under the category of collections, and that some of them fall under the sub-category of Unities).

The relationship between *is a proper part of*, on the one hand, and *is a pre-element of* and *is a proper sub-collection of*, on the other hand, may be illustrated by means of the analogous relationship between various relations of similarity. Consider the generic relation *is perfectly similar in some respect to*, which can be expressed by means of 'is PS- X to'. The relations *is perfectly similar in respect of colour to*, *is perfectly similar in respect of shape to*, are among the species of this relation - expressed in abbreviated form by means of 'is PS-colour to' and 'is PS-shape to', respectively. In general, species of *is PS- X to* are designated by expressions of the form 'is perfectly similar in respect of A to', where ' A ' is substituted by abstract nouns from a range to which 'colour' and 'shape', among others, belong (roughly, the range includes nouns expressing properties; greater precision is not required for the purposes of the present illustration).

The task of constructing a formal theory of the relation *is PS- X to* is ambiguous as between the task of characterizing the generic formal features of species of this relation, and the task of characterizing the formal features of the generic relation itself. On the former understanding, one would lay down the assumption that *is PS- X to* is transitive (for any species of this relation is transitive). On the latter understanding, by contrast, one would lay down the assumption that *is PS- X to* is non-transitive (for the generic relation itself is not transitive).

There is an analogous ambiguity regarding the task of constructing a formal theory of the relation *is a proper part of*. Assuming that this relation is generic, and that at least two relations belong to it as species (namely, *is a pre-element of* and *is a sub-collection of*), then the task might be understood as that of characterizing generic formal features of the species, or as that of characterizing formal features of the generic relation itself. I shall understand the task in the former way.

Thus it becomes clear how one may consistently claim, for instance, that the relation *is a proper part of* is transitive (in the sense that any species of the relation is transitive), and yet assert that although x is a proper part of y , and y is a proper part of z , x is nevertheless not a proper part of z . For if x is a pre-element of y then x is a proper part of y (the former relation is a species of the latter). And if y is a sub-collection of z then y is a proper part of z (again, the former is a species of the latter). But from these assumptions it neither follows that x is a pre-element of z , nor that x is a sub-collection of z . And since these are the only two species of the relation *is a proper part of* which we have held to exist, it does not follow from our assumptions that x is a proper part of z (even though we assert the transitivity of *is a proper part of* in the sense explained).

When we compare the account of the mereology of individuals offered by the theory of Unities with accounts offered by classical mereology, or neoclassical mereology, we may take these theories as sharing the above stated assumptions and definitions, with the exception of definition D5 (this definition is accepted in classical mereology and in the theory of Unities, but not generally accepted in neoclassical mereology). Keeping this difference in mind, we may use the terms involved in these assumptions and definitions in a way relatively free from controversy, and point out particularities of the theory of Unities by using terms specific to it, terms which are associated with further assumptions and definitions to which we now turn.

8.3.3 Collections and the Relation *is a proper sub-collection of*

We begin by treating the first of the two species of *is a proper part of* (and of *is a whole which corresponds to*), which we distinguish in the theory of Unities: the relation *is a proper sub-collection of*.

We note first that the formal relation between *is a proper sub-collection of* and *is a sub-collection of* corresponds precisely to that between *is a proper part of* and *is a part of*, respectively, as stated in definition D5. We may further define the relations 'collection-overlaps', 'is collection-disjoint from', and 'is a collection-sum of', in terms of 'is a proper sub-collection of', precisely in parallel to the definitions of 'overlaps', 'is disjoint from' and 'is a sum of', respectively, in terms of 'is a proper part of'.²⁸ Using these definitions, we can prove on the basis of A3-A7 and D2-D4 that collections form a system, with respect to the relation *is a proper sub-collection of*, which is isomorphic to classical mereology. For we can prove the following statements, which correspond to the axioms Simons gives for classical mereology:²⁹

²⁸ Indeed, one may supply the theory of collections with definitions which parallel all the definitions given in association with 'is a proper part of' above. However, this will not be required for our purposes; we may always use the terms defined in terms of 'is a proper part of', since the context will generally make it clear in what follows which species of the generic relation is intended.

²⁹ See Simons 1987, 37, axioms SA1, SA2, SA3 and SA24. The proofs for A15-A18 are offered as follows:

Proof of A15:

1. x is a proper sub-collection of y . [assumption]
2. x is a sub-collection of y . [by 1 and D4]
3. for all z , z is a member of x only if z is a member of y . [by 2 and D3]
4. for all y , for all x , if y is a proper sub-collection of x then for some w , w is a member of x but not of y . [by D4]
5. y is not a proper sub-collection of x . [by 3 and 4]
6. A15. [by 1 and 5]

Proof of A16:

1. x is a proper sub-collection of y . [assumption]
2. y is a proper sub-collection of z . [assumption]
3. x is a sub-collection of y . [by 1 and D4]
4. y is a sub-collection of z . [by 2 and D4]
5. for all w , w is a member of x only if it is a member of y . [by 3 and D3]
6. for all w , w is a member of y only if it is a member of z . [by 4 and D3]
7. for all w , w is a member of x only if it is a member of z . [by 5 and 6]
8. x is a sub-collection of z . [by 7 and D3]
9. for some w , w is a member of z but not of y . [by 2 and D4]
10. for some w , w is a member of z but not of x . [by 9 and 5]
11. x is a proper sub-collection of z . [by 8 and 10]

- A15 for all x , for all y , if x is a proper sub-collection of y then y is not a proper sub-collection of x .
- A16 for all x , for all y , for all z , if x is a proper sub-collection of y and y is a proper sub-collection of z then x is a proper sub-collection of z .
- A17 for all x , for all y , if x is a proper sub-collection of y then for some z , z is a proper sub-collection of y and z is collection-disjoint from x .
- A18 for any xs , for some y , y is a collection-sum of the xs .

This formal isomorphism with classical mereology allows us to assume, consistently with our assumptions regarding *is a proper part of*, that *is a proper sub-collection of* is a species of *is a proper part of* (a species which is found only in cases

12. A16. [by 1, 2 and 11]

Proof of A17:

1. x collection-overlaps y =_{def.}
for some z , z is a sub-collection of x and z is a sub-collection of y . [definition]
2. x is collection-disjoint from y =_{def.}
it is not the case that x collection-overlaps y [definition]
3. x is a proper sub-collection of y . [assumption]
4. for some z , z is a member of y but not of x . [by 3 and D4]
5. for all z , if z is a member of y then for some v , v is a collection whose only member is z , and v is a sub-collection of y . [by A6, C3 and D3]
6. for all v , for all z , if v is a collection whose only member is z , and if z is not a member of x , then v is collection-disjoint from x . [by 1 and 2]
7. if z is a member of y but not of x , then for some v , v is a collection whose only member is z , and v is a sub-collection of y , and v is collection-disjoint from x . [by 5 and 6]
8. for some v , v is a sub-collection of y and v is collection disjoint from x . [by 4 and 7]
9. A17. [by 3 and 8]

Proof of A18:

1. y is a collection-sum of the xs =_{def.}
for all z , z is a member of y if and only if for some u , u is one of the xs and z is a member of u [definition]
2. the $xs = x_1, \dots, x_n$ =_{def.}
 z is one of the xs iff z is identical to x_1 or to ... or to x_n . [definition]
3. the $xs = x_1, \dots, x_n$. [assumption]
4. for $i = 1 \dots n$, x_i has exactly $m(i)$ members, and for $j = 1 \dots m(i)$, u_{ij} is the j 'th member of collection x_i . [assumption]
5. the $zs = u_{11}, \dots, u_{1m(1)}, \dots, u_{n1}, \dots, u_{nm(n)}$ [assumption]
6. each of the zs is a Unity. [by A5, 4 and 5]
7. for some y , y is a collection of the zs . [by A6, 6]
8. for any xs , for some y , y is a sum-collection of the xs . [by 1-5 and 7]
9. A18. [8]

where, if x is a proper part of y , y is not a Unity, for a member of a sub-collection does not have proper sub-collections³⁰):

A19 for all y , for all x , if y is not a Unity, then

x is a proper part of y iff x is a proper sub-collection of y .

As for the species of *is a whole corresponding to*, associated with this species of *is a proper part of*, we make the following assumption, in the spirit of classical (and indeed neoclassical) mereology:

A20 for any xs , for all y , if the xs are sub-collections of y , then y is a

whole which corresponds to the xs iff y is a collection-sum of the xs .

In effect, A19 and A20 guarantee that our theory of collections is not merely isomorphic to classical mereology, it is an *instance* of classical mereology. It is classical mereology viewed as applying to collections of Unities (which are taken to be collections of individuals) rather than to individuals.³¹ This allows us to apply here the result of our discussion of Section 7.2, regarding the identity between a classical sum and its parts.

A21 for all y , for any xs , if y is a collection-sum of the xs , then y is

collectively identical to the xs .

It follows from A20 and from the isomorphism with classical mereology that anything which is a whole (of this species) which corresponds to the xs , is *unique*:

A22 for any xs , for all y , for all z , if the xs are sub-collections of y and y is

a whole which corresponds to the xs , and if the xs are sub-collections of z and z is a whole which corresponds to the xs , then $y=z$.

³⁰ A member of a sub-collection is, by A5, a Unity. A Unity is a single-membered collection (by A7). A single-membered collection cannot have proper sub-collections (by D4).

³¹ Note that A20 implies that collections comply, with respect to their sub-collections, with D4 of Section 8.2.1.

8.3.4 Unities and the Relation is a pre-element of

We now lay down assumptions associated with the second, and more important, species of the relation *is a proper part of* which is acknowledged by the theory of Unities. On our assumptions so far, no allowance was made for proper parts of Unities (see A19). In the theory of Unities it is assumed, however, that not only multi-membered collections, but also Unities, may have proper parts. We shall assume that only Unities can be proper parts of Unities. The species of *is a proper part of* which is found as a relation between Unities, however, has formal characteristics distinct from those of the relation *is a proper sub-collection of* (apart from the common core which they share, as expressed by D5-D11 and A9-A14). We therefore designate it by a distinct term, 'is a pre-element of'.³² Thus, the proper parts of a Unity are its pre-elements.³³

D12 x is a pre-element of $y =_{\text{def.}}$

x and y are Unities, and x is a proper part of y

A particularly important role in the theory of Unities is accorded to *immediate* proper parts of Unities (see definition D7 above). We call such parts 'elements'. Thus we have

D13 x is an element of $y =_{\text{def.}}$

x is a pre-element of y and x is an immediate proper part of y .

If each of the x s is an element of Unity y , and y is a whole which corresponds to the x s, then the x s are said to underlie y :

³² It is possible to present in connection with 'is a pre-element of' a battery of definitions paralleling those given above in association with 'is a proper part of'. This would require introducing many additional terms, which would distinguish between generic relations (associated with 'is a proper part of') and specific relations (associated with 'is a pre-element of'). However, this would be of little use in practice, and no confusion should arise, in the context, from use of the respective generic terms. (See also first footnote of the present Subsection).

³³ We are not assuming that *all* Unities have proper parts. If a Unity has no proper parts, then, of course, it has no pre-elements, and it may be described as an *atomic* Unity.

D14 the xs underlie $y =_{\text{def.}}$

each of the xs is an element of y , and y is a whole which corresponds to the xs .

Just as for any xs only one collection can be the collection-sum of the xs (if each of the xs is a collection) - for the theory of collections presented here is, as explained above, an instance of classical mereology, and uniqueness of sums is a feature of classical mereology - we similarly assume that only one Unity can be underlain by the xs (if the xs are elements of a Unity):

A23 for any xs , for all y , for all z , if the xs underlie y and the xs underlie z then $y=z$.

However, although we are assuming that for any xs , at the most one Unity is underlain by the xs , we are not assuming that at most one Unity is a whole which corresponds to those xs . To bring this point out, consider the following illustration:



Suppose B is an organism (greatly simplified as regards complexity); b_1 and b_2 are the organs of B (at time t) - where entities such as a heart or a muscle are taken to be organs of an organism; and $a_1 \dots a_6$ are the cells of B (at time t). Suppose we find it reasonable to apply the theory of Unities so as to assume that b_1 and b_2 underlie B , and that a_1, a_2 and a_3 underlie b_1 , whereas a_4, a_5 and a_6 underlie b_2 .

The cells do not underlie B , on these assumptions, for since each of them is a proper part of an element of B (i.e. one of the organs), none of them is an immediate proper part of B , and therefore none of them is an element of B (see D13 and D14). But our assumptions are compatible with the highly plausible supposition that B is nevertheless a whole which corresponds to a_1, a_2, a_3, a_4, a_5 , and a_6 . If that is the case, then B is both a whole which corresponds to b_1 and b_2 , and a whole which corresponds to a_1, a_2, a_3, a_4, a_5 , and a_6 .

Now, although as just noted it follows from our assumptions that the cells a_1 , a_2 , a_3 , a_4 , a_5 , and a_6 do not underlie B , it is consistent with our assumptions that there is some Unity A (see illustration) which is not identical to B which the cells a_1 , a_2 , a_3 , a_4 , a_5 , and a_6 do underlie (it might be suggested for instance that A is a lump of flesh which constitutes the organism and yet is not identical to it). If the cells underlie A then by D14 it follows that A is a whole which corresponds to those cells. We conclude, then, on these assumptions, that both A and B are wholes which correspond to a_1 , a_2 , a_3 , a_4 , a_5 , and a_6 , and yet $A \neq B$.

Thus, although for any xs no more than one Unity may be underlain by the xs , more than one Unity may be a whole which corresponds to the xs .

In a fully developed theory of Unities, the relation *is a whole which corresponds to* should be explicated in detail, not only as obtaining between a Unity and its elements (in which case we have a relation between a whole and parts which underlie it), but also as obtaining between a Unity and pre-elements which are *not* elements (in which case we have a relation between a whole and parts which do *not* underlie it). However, it would exceed the scope of the present work to provide such an exposition here.

The relation *is a whole which corresponds to* as obtaining between Unities and their elements, is distinguished from the relation as obtaining between collections and their sub-collections, principally in that while the latter requires that wholes be fundamentally sums, the former does not. Even if the xs are not coextensive with the ys , it is still possible both for the xs and for the ys to underlie the same Unity. We bring this distinctive feature of Unities out by means of the following assumptions.

First, we point out that it is possible for the xs to underlie y without composing y (and thus - taking into account D14 - we are denying D4 of Subsection 8.2.1):

A24 for some xs , for some y , the xs underlie y but do not compose y .

And similarly, denying a direct implication of D4, we assume:

A25 for some xs , for some y , for some zs , the xs underlie y and the zs underlie y , but the xs are not coextensive with the zs .

We do assume, however, with regard to *all* the elements of a Unity, that they not only underlie it but also compose it:

A26 for all xs , for all y , if the xs are all the elements of y then the xs underlie y and the xs compose y .

This also guarantees that assumption A5 of Subsection 8.2.1 is fulfilled, for A26 implies that

A27 if the xs underlie y then for some zs , the xs are among the zs and the zs both compose and underlie y .

Finally, it is useful to have a term by means of which to describe collections whose members underlie a Unity. We shall speak of such collections as *embodying* collections, and define as follows:

D15 x embodies y $=_{\text{def.}}$
for some zs , the zs are all the members of x , and the zs underlie y .

Our assumptions obviously entail that many distinct collections may embody one and the same Unity. This suggests an interesting perspective from which to compare the conception of a whole as a Unity with its conception as a sum. Conceiving of a whole as a sum, we imagine that the relation between a whole and its parts can be satisfactorily represented by means of a dyadic predicate ' Rxy ', satisfied in case a term designating some whole is substituted for ' x ' and a term designating any of its parts is substituted for ' y '. Conceiving of a whole as a Unity, we rather imagine the relation between a whole and its parts as represented by means of a different dyadic predicate, ' Sxy ', which is satisfied in case (if ' x ' is substituted by a term designating some whole) ' y ' is substituted by a term designating not any *part* of the whole, but rather any *embodying collection of parts* of the whole. I shall not pursue this suggestive perspective at present, however.

8.3.5 Loose Uses of 'is a proper part of'

According to the theory as outlined so far, instances of the relation *is a proper part of* fall under two species: some are instances of *is a proper sub-collection of*, and others instances of *is a pre-element of*. Thus we account for whole-part relations between collections and their sub-collections, and between Unities and their pre-elements. It is natural to wonder whether one must not account also for other apparent whole-part relations, as, for example, the relation that would seem to hold between a Unity and a collection of *some* (i.e. more than one) of its elements. Such a relation is not countenanced in our account, as developed so far.

Furthermore, as we have seen, different Unities can be simultaneously sums of precisely the same pre-elements. This means that one Unity can be at the same place at the same time as another Unity. Since two such Unities cannot share all their pre-elements (otherwise they would be identical according to A23), the following question arises: how should one describe the relations between a Unity u and some pre-element x of a superposed Unity, v , where x is not a pre-element of u .

One way in which one might introduce such relations would be by developing the theory in a way which renders all these varied types of relations as further species of the relation *is a proper part of*. My attempts so far (which I do not record here), however, indicate that this would lead to a substantially more complicated theory, with few evident advantages.

An alternative approach, which is both simpler and possessed of an intrinsic plausibility, is to render such apparent instances of *is a proper part of* as not being *bona fide* instances of that generic relation. On this account, using 'part' to refer to such relations involves a loose usage, one which should be distinguished from the strict usage associated with the relation *is a proper sub-collection of* and *is a pre-element of*.

I now sketch in outline, how such an account can be provided. One type of case may be accounted for by the simple means of introducing the notion of an *indirect part*. On the principles of the theory expounded above, a molecule m which is a part of some brick b cannot be, strictly speaking, a part of a collection of bricks, one of whose members is b . Supposing the brick to be an individual, and therefore a

Unity, the molecule is a pre-element of the brick, and therefore a part of the brick (in that it bears to the brick a relation associated with one of the two species of *is a proper part of* which we have distinguished). The brick, on the other hand, is a sub-collection of the collection, and therefore a part of the collection (bearing to the collection a relation associated with the other of the two species of *is a proper part of*).

However, the molecule is neither a pre-element of the collection (for multi-membered collections have no pre-elements), nor a sub-collection of the collection (for only bricks and collections of bricks are among the sub-collections of the collection). So the molecule is not a part of the collection. However, the molecule is a part (i.e. a pre-element) of a part (i.e. a sub-collection) of the collection of bricks, although not itself a part of the collection of bricks. This, as we explained above, does not conflict with the transitivity of *is a part of* (recall the relations *is PS-shape to* and *is PS-colour to*). I now propose to describe the molecule as an *indirect* part of the collection, defining as follows:

D16 x is an indirect part of $y =_{\text{def.}}$

x is neither a sub-collection of y nor a pre-element of y

but for some z ,

x is either a sub-collection of z or a pre-element of z and

z is either a sub-collection of y or a pre-element of y .

Other problematic circumstances seem to be found only in cases where for some xs , several distinct entities, y , z , etc., are each a whole which corresponds to the xs . For example, if each of the xs is a Unity, then on the one hand there is y , the collection of the xs . On the other hand, there may be a Unity z , which is underlain by the xs ; furthermore, there may be another Unity, w , which is underlain by elements which are in turn underlain respectively by elements from among the xs ; etc.

In order to deal with such cases, I propose the following adaptation of the notion of superposition (taking into account our distinction between two species of the relation *is a part of*):³⁴

³⁴ 'Superposition' was defined above at the beginning of Section 4.3.

D17 u is superposed with $v =_{\text{def.}}$

the xs compose u and the xs compose v

(either in the sense of 'compose' associated with *is a pre-element of* or in the sense associated with *is a sub-collection of*).

We may now introduce the notion of a *vicarious part*. For example, suppose Statue is a clay statue, and suppose Lump is the lump of clay which constitutes it (at time t). The theory of Unities is consistent with the claim that Lump is not identical to Statue (as well as with the claim that they are identical). Supposing they are not identical, the resources of the theory of Unities allow us to account for this state of affairs in various ways. An outline of one such possible account is as follows (this account is obviously over-simplistic, but it is only meant to illustrate the principle).

We assume Lump is a Unity, the elements of which are so-and-so many millions of clay particles. Call those particles 'the ps '. Statue, on the other hand, is also a Unity, with elements Arm1, Arm2, Leg1, Leg2, Torso, Neck, and Head. Each of these elements of Statue, in turn, has some of the ps as its elements.

Arm1, on this hypothesis, is a part of Statue (for it is an element of it). It is not a part of Lump, however, because Lump being a Unity, its only parts are Lump itself and its pre-elements, which are the clay particles and *their* pre-elements. We may, however, describe Arm1 as a *vicarious part* of Lump. We explain this notion as follows:

D18 x is a vicarious part of $y =_{\text{def.}}$

x is neither a sub-collection of y nor a pre-element of y

but for some z , such that y is superposed with z ,

x is either a sub-collection of z or a pre-element of z .

Accounting for cases which do not fall under the rubric of *bona fide* instances of the relation *is a proper of* in this way, i.e., as instances of the relations *is an indirect part of* and *is a vicarious part of*, would seem to satisfy the intuitive pull towards viewing, for example, a molecule as a "part" of a collection of bricks, and Statue's Arm1 as a "part" of Lump. Strictly speaking, we should not describe these as parts; but loosely speaking we may, for the relations between the molecule and the collection, and between Arm1 and Statue, are in obvious ways derivable from, and

associated with, relations which are strictly speaking relations between parts and wholes.

Chapter 9

Further Elaborations and Applications of the Theory

Section 9.1

Perspectives on the Theory of Unities

9.1.1 *Conditions of Identity*

We are familiar with assumptions according to which the identities of comprising entities are bound up with the identities of entities they comprise. These assumptions provide us with conditions for the identity of a comprising entity stated with reference to the identities of the comprised entities. These conditions may be stated, noting in each case in brackets the relation which establishes the identity dependence, as follows. In the case of classes, we have:

Identity (via *is a member of*) for Classes

for all u , for all v ,

if u and v are classes, then $u = v$ iff

for all x , x is a member of u iff x is a member of v

In the case of classical sums, we have the following similar assumption:

Identity (via *is a part of*) for Classical Sums

for all u , for all v ,

if u and v are classical sums, then $u = v$ iff

for all x , x is a part of u iff x is a part of v

These assumptions express a feature of classes and classical sums known as their *extensionality*. Neoclassical mereological principles, by contrast with those

governing classes and classical sums, do not imply the extensionality of the comprising entities they countenance. For with regard to neoclassical sums the identity of parts is only necessary, and not sufficient, for the identity of their sums:

Identity (via *is a part of*) for Neoclassical Sums

for all u , for all v ,

if u and v are neoclassical sums, then $u = v$ only if

for all x , x is a part of u iff x is a part of v

It is by giving up the assumption that the identity of parts is sufficient for the identity of their sums, that neoclassical theory allows sums to be introduced which are superposed with one another.

A condition of identity for Unities may be stated which is analogous to the one applying to classes and classical sums (similarly contrasting with the condition which applies to neoclassical sums):

Identity (via *is an element of*) for Unities

for all u , for all v ,

if u and v are Unities, then $u = v$ iff

for all x , x is an element of u iff x is an element of v

We see then that Unities share with classes and classical sums the feature of extensionality.

The relations *is a member of*, *is a part of*, and *is an element of* are relations, in the case of each type of comprising entity, between *each* of the comprised entities and the comprising entity. Identity determination between comprised and comprising entities runs, however, not only through these relations but also through relations which do *not* hold between each of the comprised entities and the comprising entities. Regarding wholes, 'make up' was the pre-theoretical term we used for such relations. In the case of classical and neoclassical mereology this^{is} interpreted as the relation *compose*. In the theory of Unities, as the relation *underlie*. I call the analogous set-theoretical relation *form*. We have conditions of this type for classes and classical sums as follows:

Identity (via *form*) for Classes

for all u , for all v , for any xs , for any ys ,
if u and v are classes, and the xs form u and the ys form v ,
then $u = v$ iff
for all z , z is one of the xs iff z is one of the ys

Identity (via *compose*) for Classical Sums

for all u , for all v , for any xs , for any ys ,
if u and v are classical sums, and the xs compose u and the ys compose v ,
then $u = v$ iff
for all z , z overlaps one of the xs iff z overlaps one of the ys

In the case of neoclassical sums, there is a condition similar to the one just stated for classical sums, the difference being that in the neoclassical case a conditional replaces the biconditional, as follows:

Identity (via *compose*) for Neoclassical Sums

for all u , for all v , for any xs , for any ys ,
if u and v are neoclassical sums, and the xs compose u and the ys compose v ,
then $u = v$ only if
for all z , z overlaps one of the xs iff z overlaps one of the ys

We see then that classes compare in this respect with sums, both classical and neoclassical, as follows. For any xs to determine the same class as do the ys , the xs have to be *identical* to the ys . By contrast, for any xs to determine the same sum (classical or neoclassical) as do the ys , the xs have to be *coextensive* with the ys .

In the case of Unities, the determining relation requires neither identity nor coextensivity. That is, for any xs to determine the same Unity as do the ys , the xs neither have to be identical to, nor coextensive with, the ys . By contrast with the case of *neoclassical* sums, however, the extensionality of Unities implies that *if* (to emphasize: 'if', not 'if and only if') the xs are severally identical to the ys then the Unities they underlie are identical:

Identity (via *underlie*) for Unities

for all u , for all v , for any xs , for any ys ,
if u and v are Unities, and the xs underlie u and the ys underlie v ,
then $u = v$ if
for all z , z is one of the xs iff z is one of the ys .

The contrast between the different ways in which the identity of comprised entities determines the identity of comprising entities, in connection with classes, classical (and neoclassical) sums, and Unities, suggests that we should distinguish these as three general types of comprising entity. In Part I it was explained that modern theorizing about comprising entities is characterised by a distinction between two basic types of comprising entity: that of distributive classes, and that of collective classes. Classes, as featuring in set theory, are a development of the former type. Sums, as featuring in both classical and neoclassical mereology, are alternative developments of the latter type. The fundamental nature of the contrast between those two basic types of comprising entity is exhibited by the contrast between the identity-determining relations explained above.

Again, in the case of classes, the xs and the ys may form the same class only if the xs are identical to the ys . In the case of sums, the xs and the ys may compose the same sum only if the xs are coextensive with the ys . The fact that in the case of Unities, neither of these requirements apply, indicates that the notion of a Unity should be taken as that of a third basic type of comprising entity, alongside classes and sums.

9.1.2 *Analogy with Other Relations*

In order to grasp more clearly what the relation *underlie* involves, and what sort of identity dependence it establishes between elements of a Unity and the Unity they underlie, contrasting this dependence with those established by the relations *form* and *compose*, it will be helpful to consider a similar contrast which occurs in relations which are not those of parts to wholes. A rather close analogy to the contrast between *underlie* and *compose* can be found among relations between

geometrical points and figures which they specify. In order to avoid complications arising from the nature of abstract entities, I shall consider dots on a blackboard instead of genuine geometrical points.

If three dots are marked on an otherwise clear blackboard, then there is one and only one circle which could be drawn through those dots (ignoring the fact that the dots have a finite size, and therefore that they do not specify in strict precision the course through which the circle will run). Let us think of this as an invisible circle which actually runs through the dots (perhaps it is only visible through infra-red sensitive goggles). There are infinitely many different (invisible) circles which run through any two of these dots, but only one which passes through all three of them. The three dots might thus be said to specify a single circle.

Now suppose not three but ten dots are marked on the blackboard, such that a single circle could be drawn through them all. Clearly, any three of the ten dots specify the same circle (which passes through them all). Moreover, any four dots, or indeed any n dots (where n is greater than 2) of those marked would serve the same purpose. A relation which may be called 'circle-specify' thus holds between any three or more of the dots and the circle that is drawn through them.

We can now see that the relation between the circle and ten dots which lie on it is analogous to the relation between a Unity and its elements. The relation *circle-specify*, as holding between dots and circles, is analogous to the relation *underlie*, as holding between elements and Unity. Just as several of the Unity's elements (not necessarily all) may underlie it, so several of the dots, (not necessarily all ten of them) may circle-specify the circle. And just as non-coextensive collections of elements may underlie the same Unity, so non-coextensive collections of dots may be such that the members of either of them circle-specify the same circle.

It must be emphasized, though, that the relation *circle-specify*, as holding between dots and circles, is only *analogous* to the relation *underlie*, and should by no means be assumed to be an *instance* of it. The relation *circle-specify*, indeed, is not a relation between parts and a whole. The dots are made of chalk. The circle, we have

assumed, is invisible, and has no parts which are made of chalk.³⁵ The relation *underlie*, by contrast, is a relation between parts and a whole which they make up.

Now, consider by contrast the same ten dots as specifying not circles, but rather polygons whose vertices coincide with dots. Taking any three of the dots, there is clearly one and only one polygon all of whose vertices coincide with those dots, and this polygon, of course, is a triangle. The three dots may then be said to specify that triangle. Let us call the relation of specification between dots and polygons, thus conceived, 'polygon-specify'.

It is clear that if the *xs* and the *ys* are any dots among then ten dots on the blackboard, the *xs* polygon-specify the same figure as do the *ys* if and only if the *xs* are identical to the *ys*.³⁶ Thus the relation *polygon-specify*, as holding between dots and polygons, is analogous to the relation *form*, as holding between members of a class and the class.

To construct an analogy with *compose*, as holding in connection with classical sums, consider not the dots individually, but rather collections³⁷ of dots, as specifying polygons. Thus suppose there are pairs, triplets, etc. of dots. If, for example, *x* is a pair of dots, and *y* is a triplet of dots, then *x* and *y* can be taken as specifying a polygon as follows: there is one, and only one polygon, such that any and only those dots that are members either of *x* or of *y* coincide with vertices of the polygon. The relation between collections of dots and polygons they specify in this way may be called 'co-polygon-specify'.

Inspection reveals that if the *xs* are one or more collections of dots, and the *ys* are one or more collections of dots, then the *xs* co-polygon-specify the same figure as the *ys* if and only if the *xs* are coextensive with the *ys*. The relation *co-polygon-specify*, as holding between collections of dots and polygons they specify, is thus clearly analogous to the relation *compose* as featuring between parts and their classical sum.

³⁵ However, even if both the dots and the circle were made of chalk, the ten dots obviously would obviously not make up the circle, and so the circle would not be a whole which corresponds to the ten dots.

³⁶ Again, we say the *xs* are identical to the *ys* in case for all *x*, *x* is among the *xs* iff *x* is among the *ys*. I have sometimes used the expression 'severally identical' above to express such identity claims holding between the *xs* and the *ys*.

³⁷ I use 'collection' here in the more general sense presented in Part I, rather than in the narrower sense pertaining to the theory of Unities.

Finally, if one wishes to construct an analogy with *compose*, as featuring in connection with neoclassical sums, one may modify the last example by assuming that each collection of dots might specify not one, but two or more invisible polygons which are superposed with one another (we might take each such polygon to be visible through different types of goggles - infra-red sensitive goggles, ultra-violet sensitive ones, etc.).

Again, it should be emphasized that each of the relations presented between dots and figures are to be taken as analogous to, rather than as instances of, the corresponding comprising relations.

Four ways have been noted - according to which dots, or collections of dots, on the blackboard can be construed as specifying figures. In each case, the identities of the dots, or of the collections of dots, are seen to determine the identity of a figure in a particular way.

Focusing on the conditions which comprised entities necessarily meet, if the entity which comprises them is to be the same, the cases are seen to fall into three types, corresponding to *circle-specify* (as holding between dots and circles), *polygon-specify* (as holding between dots and polygons), and *co-polygon-specify* (as holding between collections of dots and polygons). These relations exhibit patterns of identity determination precisely analogous to those exhibited by *underlie* (between elements and their Unities), *form* (between members and their classes), and *compose* (between parts and their sums - classical or neoclassical) respectively.

9.1.3 *The Character of Unities*

Let us return to a point raised briefly in 8.2.1. Consider again the house which has been built out of bricks, $b_1 \dots b_n$ (abbreviate ' $b_1 \dots b_n$ ' to 'the bs '). Suppose beside the house there has been left a pile of unused bricks. If we describe the house as a sum composed by the bs , then the notion of a sum guarantees that while the bs are parts of the house, the bricks in the pile are not parts of the house. If the $bs+cs$ are all the bricks of the house *and* the bricks in the pile, then if there is a sum of the

$bs+cs$, it is clear that this sum is not identical to the sum of the bs . For there are entities which overlap the sum of the $bs+cs$ which do not overlap the sum of the bs .

However, if the house is described as a Unity underlain by the bs , then there is nothing in the assumptions so far made regarding Unities which would preclude the possibility that not only the bs , but also the bricks in the pile, are elements of the house. It might be thought that if that is so, the notion of a Unity and of the relation *is an element of* cannot appropriately be considered as providing an account of our notions of whole and part. For it might be thought that if the identity of a whole is determined (by identifying parts which make it up) then it must be possible to determine on this basis regarding any entity whether or not it is a part of the whole. In conflict with this, the theory of Unities seems to allow us to determine the identity of the house without enabling us to deduce from this that the bricks lying beside it are not parts of it.

Such an objection, however, is based on a misunderstanding. True, the assumption^{that} the house is composed by the bs implies that the house is not composed by the $bs+cs$, whereas the assumption that the house is underlain by the bs does not imply that the house is not underlain by the $bs+cs$. However, in either case the respective assumptions derive from our understanding of the concept of a house. It is on the basis of this understanding that one is led to claim either that the house is composed by the bs or that it is underlain by the bs . Furthermore, it is precisely on the basis of this understanding that one is led to claim, either that the house is *not* composed of the $bs+cs$, or that it is not underlain by the $bs+cs$.

The only difference between the cases is that, whereas in case we conceive of the house as a sum we may arrive at the claim that the house is not composed of the $bs+cs$ not only directly on the basis of the concept, but also indirectly, on the basis of the assumption that the house is composed of the bs (where this in turn is derived directly from the concept), we are not given such an indirect inferential path in case we conceive of the house as a Unity. This, however, in no way weakens our justification for thinking that the house is not underlain by the $bs+cs$. The claim - given the assumption that a house is a sum - that the house is not composed by the $bs+cs$, rests on precisely the same foundation as the claim - given the assumption that a house is a Unity - that the house is not underlain by the $bs+cs$, and so in this respect both claims are justifiable to the same degree.

Far from being a disadvantage of the theory of Unities, the lack of entailment between the claim that the *bs* underlie a Unity and the claim that the *bs+cs* do not underlie a Unity is a feature of the theory which exhibits striking advantages in a variety of types of cases. I shall give two examples.

9.1.3.1 *Vagueness in Sortal Concepts*

It is commonly observed that many of our concepts for types of entity are vague, in the sense that the concept seems not to determine clearly with regard to some things, whether or not they are parts of the entity. For example, the concept of a battlement (in the sense of ^acertain kind of part of a castle)³⁸ would often seem not to determine whether a certain stone which is a part of the castle is or is not also a part of the battlement. One is likely to be unsure where the battlement ends and the rest of the castle begins.

If a battlement is considered to be a sum of stones, then the vagueness in the concept of a battlement reads as a vagueness as to which stones compose the battlement. The concept does not allow us to say determinately with regard to *any* stones, that they compose the battlement. But this in turn becomes a vagueness as to the *identity* of the battlement.

For suppose the *xs* and the *ys* are stones, where some of the *ys* are not among the *xs*, and suppose sums exist both of the *xs* and the *ys*, so that a sum of the *xs* is distinct from a sum of the *ys*. Suppose the difference between the two sums is clearly visible; the *ys* include ten extra stones constructed to the left, adjacent to the *xs*. Suppose we are unproblematically able to identify these two sums. However, since the vagueness in the concept makes us doubt whether the battlement is identical to the sum of the *xs* or to the sum of the *ys*, then, since the sums are distinct, we are in doubt about the identity of the battlement. It turns out then that on the conception of a battlement as a sum, the vagueness in the concept of a battlement becomes a vagueness as to which entity the battlement is identical to.

³⁸ *The Concise Oxford Dictionary of Current English* gives the following definition of 'battlement': "Indented parapet ...; parapet & enclosed roof."

Now consider the alternative conception of a battlement as a Unity. Of course, such a change in conception does not make the vagueness in the concept of a battlement disappear. However, it can readily be seen that now this vagueness does not lead to vagueness regarding the identity of the battlement. Rather, the vagueness is confined merely to the question, regarding some of the battlement's peripheral stones, whether these stones are among the elements of the battlement. Conceiving of the battlement as a Unity, in this case, we are admittedly still unclear about the status of some stones as elements of the battlement, but we are not unclear about the identity of the battlement: we are not unclear, as before, as to whether the battlement is one or the other of two distinct, existent wholes. A vagueness affects the identity of the complete embodying collection of stones, but not the identity of the embodied Unity itself.

The reason for this is that when one doubts with regard to some stones whether they are among the elements of the Unity, one may still be clear with regard to *some* stones that they underlie the battlement. If the *xs* are such stones, where each of them is *determinately* (rather than vaguely) an element of the battlement, and the *xs* underlie the battlement, then the identity of the battlement is fixed as a Unity underlain by the *xs*. Whether or not for some *ys* (where not all of the *ys* are among the *xs*) the *ys also* underlie the battlement in no way involves a recasting of the battlement's identity. The remaining vagueness pertains only to the question whether the additional stones are to count also as elements of the same Unity.

Pre-theoretically we do not seem to be in doubt as regards the identity of the battlement, although we may well be in doubt as to whether a certain stone is one of its parts. We do not think that there are two, or perhaps a hundred different entities, largely overlapping one another, any of which might be the battlement. If someone points to the battlement of the castle over there, I am not inclined to ask her which of the many sums that I am able to identify she takes to be the battlement. It is clear that the conception of wholes as Unities is better suited to capturing common sense identificational practice in this respect.

An account on the lines presented here regarding the case of the battlement applies as a general treatment of the omnipresent cases in which Unger's so called

problem of the many arises.³⁹ Reflection on the received scientific portrayal of macroscopic concrete entities as constituted by swarms of micro-particles leads to viewing the former much like dust-clouds or puffs of smoke, entities which do not display sharp boundaries. At the fringes of any concrete macro-entity, according to this portrayal, there are micro-entities with regard to which it seems to be a vague matter whether they are, or are not, parts of the macro-entity.

Suppose an entity s is constituted by such a swarm of particles, where the ps are particles that are definitely parts of the swarm, and the qs , that is q_1, \dots, q_n , are particles, whose status as parts of the swarm is indeterminate.⁴⁰ If we suppose that the ps and any one of the qs (taken together) have a sum, then any such sum seems to have equal claim to being the entity s . This is so, that is, *if* we assume that s is a sum of some particles. In that case, there would seem to be no fact of the matter as to *which* of the n entities (i.e. sums of particles) is identical to s . Thus the identity of s is rendered vague.

The problem with the conception of wholes as sums, in this context, is that it compels us to assume that the claim that s is a whole which corresponds to the ps and q_i is incompatible with the claim that s is a whole which corresponds to the ps and q_j (where $i \neq j$). If we assume only one of these wholes exist, then there seems to be no hope of explaining *why* this one rather than the other exists - for there is no difference between the two cases which would reflect on the abilities of the respective parts - the ps and q_i , on the one hand, and the ps and q_j , on the other - to have a sum. If, alternatively, we assume that both wholes exist, we are faced with the need to decide which of them is s , leading us to view the identity of s as vague.

According to the conception of wholes as Unities, by contrast, the two claims (in the last paragraph) *are* compatible with one another. Deciding on one of them indeed decides the identity of s , but this does not conflict with deciding *also* on the other, for the latter decision does not compel a change in the decision concerning the identity of s .

³⁹ See Unger 1981.

⁴⁰ This must not be interpreted as suggesting that we can determine a precise boundary between particles that are determinately parts of the swarm, and parts that are not determinately parts of the swarm. We may assume nevertheless that for some xs , such that the xs are most of the microparticles that are *prima facie* candidates for being parts of the swarm, those xs are *determinately* parts of the swarm. We may take the ps as any such xs .

David Lewis surveys a number of responses to this difficulty.⁴¹ According to one proposal (due to Lowe), each of the n sums is an entity which constitutes s , but constitution, on this view, is distinct from identity, and there is no contradiction in assuming that many distinct entities may constitute a single entity. According to a second proposal, s is assumed to be a vague entity, and each of the n sums is taken to be a "precisification" of s . A third proposal (based on van Frassen's method of supervaluations), like the second one, concedes that there is no fact of the matter as to which of the n sums is identical to s , and yet, by contrast with the second proposal, argues that the claim that one of those sums is identical to s is true nevertheless, if this claim is interpreted according to the supervaluational conception of truth. According to a fourth proposal (following Geach), the n sums are different particle-sums, but the same swarm; identity is taken to be relative to sortal term. And finally, on a fifth proposal (contributed by Lewis himself, on the basis of Armstrong's notion of partial identity), identity admits of degrees, and the n sums are "almost" identical to one another, and s is as it were almost a single entity (and almost of determinate identity).

Limitations of space do not allow me to discuss these responses in detail. I shall only add the following brief comment. These views have it in common that they attempt to account for the identity of an entity with indeterminate boundaries (e.g. a cloud) in terms of the identities of entities with determinate boundaries (e.g. precisely delimited swarms, or \sum_i^S of particles). The conception of wholes as sums compels them to seek such an account. For it takes wholes to ^{be} entities which are *coextensively determined*, so that the x s cannot make up the same u as the y s unless the x s are coextensive with the y s.

The result is that the identity of a whole becomes inseparably bound up with the identity of its boundary. But the case (of the cloud, as of other macroscopic entities) does not allow them to locate a *single* precise entity, so they invariably end up explaining the identity of the imprecisely delimited entity, which on the face of it is a single entity, in terms of many almost perfectly overlapping precisely delimited entities. The conception of wholes as Unities does not bind the identity of

⁴¹ See Lewis 1993.

a whole with the identity of its boundary (unless the whole happens to have a clearly identified boundary), and so is not in the predicament of these theorists.⁴²

9.1.3.2 *Partly Perceived Entities*

We often perceive entities which are partly hidden from view. For example, suppose Tibbles the cat is sitting in the doorway, in such a way that its tail cannot be seen. Now, if we are unfamiliar with the cat, we might entertain a doubt as to whether indeed it has a tail. Suppose we consider the cat as having the following parts: Head, Neck, Torso, Leg1, Leg2, Leg3, Leg 4, and possibly Tail.

If we take the cat to be a sum (whether classical or neoclassical) of its parts, then the following difficulty appears. It would seem plausible to assume that the visible parts of the cat (i.e. the first seven in the list above) have a sum, whether or not Tail exists. It is not clear whether an assumption to the contrary, namely, that a cat which has a tail does not have a part which includes everything but the tail, could be satisfactorily motivated.⁴³ Furthermore, this assumption would imply that we are not in a position to judge whether the visible parts of the cat do or do not have a sum, unless we know whether or not the cat has other parts, outside the range of our vision. This would mean that perceptual experience is usually unable to justify the claim that a whole made up of certain parts exist, assuming that whole is a sum, for we are usually not in a position to see enough of the parts (usually you see only one side of the cat you are looking at; and in any case, its internal organs are hidden from view).

⁴² Interestingly, the constraints of the conception of wholes as sums affect even Lowe's account, which seems to come closest among currently existing views to breaking loose with it. His account still presupposes that wholes are fundamentally sums, but allows that they need not be determinately one sum or another. If the *xs* are not coextensive with the *ys*, a whole *u* may be *indeterminately* a sum of both. Whether he is right to assume that this avoids rendering the identity of the whole vague, I shall not go into. In any case it is understandable that Lewis should complain about Lowe's account, that it transfers the original problem from a problem about constituted concrete entities to a problem about constituting concrete entities. See Lewis 1993, 25-6; Lowe 1982.

⁴³ Van Inwagen makes such an assumption to the contrary in van Inwagen 1981. Simons offers some criticism of this view in Simons 1987, 121. I consider the view in some more detail in *Subsection 9.3.1*.

If we assume, then, that the visible parts of the cat have a sum, whether or not Tail exists, then it would seem that we are in a position to see and to identify this sum. However, we are not in a position to say whether what we see, when we see the sum, is identical or not to the cat.⁴⁴ For if Tail exists, then the cat is not a sum of its visible parts, but rather a sum of those parts plus Tail. This means that we are uncertain about the identity of the cat - we cannot say whether or not it is identical to something which we see, namely, the sum of its visible parts. On the conception of wholes as sums, doubt as to whether an entity has parts which are hidden from view becomes doubt regarding the identity of that entity.

Again, this result is at odds with common sense. We are often in doubt whether an entity which we see has parts which we cannot see. This, however, does not lead us to doubt whether we have succeeded in identifying the entity. If we turn to the conception of wholes as Unities, we find that here too, as in cases associated with vague sortal concepts, this conception contrasts with the former one as being in harmony with common sense.

For if we take the parts of the cat to be elements, and the cat to be a Unity, then we may claim that the visible elements of the cat underlie the cat, whether or not the cat is also underlain by elements some of which are not visible. Thus it is both the case that the visible elements of Tibbles underlie Tibbles, and (if Tail exists) that the visible elements *plus* Tail underlie Tibbles. The fact that either of two distinct collections of elements embodies a Unity is consistent with their embodying one and the same Unity. Therefore, doubt as to whether the cat has a tail or not does not lead, on the conception of a cat as a Unity, to doubt regarding the identity of the cat.

⁴⁴ This consequence remains, even if we assume that Tibbles is a non-unique sum, i.e. that it is a sum which is superposed either with a sum of the visible parts or with a sum of those parts plus Tail (such an assumption is made, for example, by Simons 1987, 112-121, and by Lowe 1989, 84-96). For suppose there is a sum of the visible parts, and, if Tail exists, a sum of the visible parts plus Tail. And in addition, there is Tibbles, a sum which is superposed with the former if Tail does not exist, and with the latter if Tail does exist. Then we might be seeing two distinct entities which are superposed with one another and are indiscernible from one another (more precisely, they are *presently* indiscernible; see 6.2.2), in the former case, or only one in the latter case. However, without knowing whether Tail exists, we cannot know which of the two possibilities is the case at hand. It is still the case then, that if we see a sum of the visible parts of Tibbles, we are in doubt as to whether this sum is identical to Tibbles or not.

9.1.3.3 *The Analogy between Elements and Properties*

The peculiar type of determinative relation which elements bear to the Unity they underlie has a particularly interesting analogue in the relation between properties and a concrete entity in which they inhere. Noting this analogy makes us further realize that the formal features of the relation *underlie* are not at all a strange special case in the realm of determinative relations. It would take us far beyond the scope of the present work to explore the issues raised by this analogy in any detail. However, relying on some simplifying assumptions, (assumptions which are not implausible, but which will not be argued for here), the basic point may be sketched briefly. Let us assume

- (1) 'Property' can be taken in a broad sense, so that whatever is designated by any monadic predicate (of the lowest logical type) which *prima facie* designates either a non-relational or a relational property, is considered to be a property.⁴⁵

Furthermore, let us assume

- (2) If x is an individual belonging to a certain broad class of individuals, x may be identified (in the actual world) with reference to several properties (in the broad sense of 'property'), such that x and only x has all those properties.⁴⁶

⁴⁵ On a view which takes there to be an ontological category of properties, whether conceived of as universals (roughly, entities which may be in more than one place at one time) or as tropes - a view with which I am sympathetic - assumption (1) is not very plausible, if it is assumed that 'property' is to apply only to members of this ontological category. Armstrong, for example, in *Universals and Scientific Realism*, convincingly criticises semantic arguments for the existence of universals, and urges that the use of a predicate in true statements provides poor evidence for the existence of a universal which is designated by the predicate (see e.g. Armstrong 1978, vol.1, 64). However, if in cases where the predicate does not designate such a real existent we may assume that the predicate can be described as designating a logical construction out of such real existents, we may take 'property' in a broad sense as covering not only the real existents but also such logical constructions.

⁴⁶ The assumption that individuals have essential properties, such that those properties may serve to identify individuals in any possible world in which they exist, is, of course, very controversial (see Kripke 1972, Lecture I). In (2), however, it is only assumed that individuals have properties, not necessarily essential ones, which may serve to identify them in the *actual* world. This does not seem to be controversial.

Thus, for example, it is reasonable to assume that if Socrates occupied a region of space s (somewhere in Athens) at time t (some time in 400BC), then Socrates can be identified as the unique individual which has both of the properties *occupies region s at time t* and *is a man*. Let us say that if the F s are properties with reference to which x can be identified, then the class whose members are (all and only) the F s is an *identifying class* of x . Let us also say, in such a case, that the F s *determine the identity* of x .

Let us assume further

- (3) There are at least two identifying classes of x , such that no property belonging to one of the classes belongs to the other.

For example, we may assume that the class whose members are the properties *is an Athenian philosopher* and *is executed in 399BC* is an identifying class of properties of Socrates. Similarly, we may assume that the class whose single member is the property *is an inventor of the maieutic educational method* is an identifying class of properties of Socrates.

Finally, let us assume

- (4) If C is an identifying class of properties of x , and if F is a property of x which does not belong to C , then the class whose members are F and the members of C is an identifying class of x .

Thus, for example, assuming the class whose members are the properties *is an Athenian philosopher* and *is executed in 399BC* is an identifying class of properties of Socrates, and assuming that *is snub-nosed* is a property of Socrates which does not belong to the latter class, we have it that the class whose members are *an Athenian philosopher, is executed in 399BC* and *is snub-nosed* is an identifying class of properties of Socrates.

On these assumptions, a clear analogy can be drawn between the relation *determine the identity of*, as holding between properties and a concrete entity, and the relation *underlie*, as holding between elements and a Unity.

Just as some of the elements of a Unity, typically not all, may underlie the Unity, so some of the properties of a concrete entity, typically not all, may determine the identity of the entity. And just as both the x s and the y s may underlie

the same Unity, even if they are not coextensive with one another, so the *F*s may determine the identity of an entity and the *G*s may determine the identity of the same entity, even if the *F*s are not coextensive with the *G*s.

We noted above that the conception of individuals as Unities allows us to provide a simple account for the phenomenon whereby we may identify partly hidden individuals. We are, of course, able to identify individuals perceptually, even if we perceive very few of the individual's (perceptible) properties. There is a clear analogy between the perceptual identification of an individual notwithstanding an only partial perception of its properties and the perceptual identification of an individual notwithstanding an only partial perception of its (concrete) parts. Taking individuals to be underlain by their parts (thus taking the individuals to be Unities and their Parts to be elements), and noting the similarities between *underlie* and *determine the identity of* (where the latter holds between some of an individual's properties and that individual), helps us account for that remarkable analogy.

Section 9.2

Applying a Theory of Unities

9.2.1 *Sketch of a Programme*

I have already noted that I take the intended interpretation of the theory of Unities as a theory of concrete individuals (in what follows we speak of concrete individuals simply as individuals). A theory of Unities is developed, therefore, with the principal aim of applying this theory to the study of individuals. I propose as the first step, in developing the theory of Unities in application to individuals, the following simple and natural one:

PA1 for all u , if u is a individual, then u is a Unity

(where 'PA' stands for 'Principle of Application'). The theory of Unities is meant to provide a certain conceptual tool for accounting for various aspects of the nature of individuals. On the assumption that some concrete entity is an individual, it will be treated in terms of the theory as a Unity. It should not be expected, however, that such a theory can provide conclusive criteria on the basis of which we might decide whether some particular entity is an individual, or whether some particular type of entities is a type of individuals. Nevertheless, the theory might offer us a range of considerations relevant to deciding on such questions.

For example, if considerations independent of the theory lead us to the view that a car is (collectively) identical to its proper parts (wheels, motor, body, etc.), then since a Unity is not identical to its elements, the proper parts of the car cannot be related to the car as elements to a Unity. Assuming that we take individuals to be Unities, this will lead us to the conclusion that a car is not an individual. However, unless such considerations external to the theory of Unities itself are brought to bear, the theory will be silent on whether the car is or is not an individual.

A useful, fully developed theory of Unities can only be elaborated in conjunction with a detailed study of individuals, and in particular, aspects of individuals which pertain to relations between parts and wholes which they

exhibit. This is not the task of the present work. Nevertheless, I do want to suggest strongly that the theory of Unities is capable of fruitful application in the study of individuals, and that it can be developed in ways required for such a theoretical purpose. To do this, I propose what seem to me to be intuitively appealing principles which establish initial steps in the application of a theory of Unities. By doing this, however, I do not wish to imply that I take these principles to be essential, and that no alternative first principles of application might be subscribed to. As the title of the present work suggests, discussion here constitutes a view *towards* a theory of Unities, and aims at sketching a *programme* of research involving the notion of a Unity.

With these qualifying comments in mind, let us look at several additional first principles guiding the way in which the theory is applied (and is further elaborated, in the course of such application).

Suppose u is an individual. Of course, this individual is a whole. According to PA1 u is a Unity. If it has proper parts which are themselves individuals, it follows that those proper parts are also Unities, and so according to D12 above they are pre-elements of u . That is,

PA2 for all u and for all x , if u is an individual and x is an individual, and x is a proper part of u , then x is a pre-element of u (that is, x is either an element of u or a proper pre-element of u)

For example, suppose we assume that a certain man m is an individual. And suppose we assume that a certain heart h is an individual, and is a proper part of m . According to principles PA1 and PA2 it follows that h is a pre-element element of the Unity m . Similarly, if a certain cell c is an individual, and c is a proper part of m , then c too is a pre-element of Unity m . Furthermore, in view of the transitivity of *is a pre-element of*, c is also a pre-element of Unity h .

To help us determine whether or not a pre-element of an individual is an element of that individual (rather than a proper pre-element of the individual), we note that the intransitivity of *is an element of* implies that an element of an element of u cannot itself be an element of u . We have, therefore,

PA3 for all x , for all u ,
 if x is a pre-element of u then x is an element of u if and only if x is an
 immediate proper part of u

Applying PA3 to our example of a man and his proper parts, we conclude that the cell c is not an element of m (and so it is a proper pre-element of m). Furthermore, if we assume that no individual exists which is a proper part of the man and of which the heart is a proper part, then we conclude on the basis of PA4 that the heart h is an element of the man m . On the assumptions made so far, however, the theory of Unities is silent as to whether there is an individual - say the man's torso - which is a proper part of the man and of which the heart is a proper part.

Now, suppose we have identified several parts of an individual as being all its elements. The question we now face is, which of those elements are such as to underlie the Unity (or, to put the question in different terms, which collections of elements are collections which embody the Unity). An obvious principle of some relevance to this question is the following:

PA4 for any xs , for all u ,
 if the xs are all the elements of u , then the xs underlie u

It may now be concluded on the basis of PA4 that if we identify a great number of cells, the cs , as all the elements of heart h , then the cs underlie h . PA4, however, tells us nothing as to whether, for example, all those cells but one also underlie the heart, or whether a certain collection of cells to which 98% of the heart's cells belong is a collection which embodies the heart.

To deal with the question whether some incomplete collection of elements⁴⁷ of a Unity embodies the Unity (or equivalently whether the members of such a collection underlie the Unity), a much more powerful principle than PA4 is required. One promising idea in this connection is to interpret conditions under which a Unity survives as evidence for determining which are the embodying collections of elements. More specifically, if a Unity survives the loss of an element at t , then the remaining elements underlie the Unity after t (according to PA4). The proposal, then, is as follows: if the remaining elements are in much the same

⁴⁷ I say that a collection of elements of u is 'incomplete' iff some element of u is not a member of the collection. Otherwise the collection is said to be 'complete'.

condition (as regards their properties and relations to one another) after t as before t , then they can be assumed have been underlying the Unity *before* t , and not only after.

Let us say that when the condition of certain elements, the xs , does not change in any way pertinent to their capacity to underlie a Unity, those elements are *not modified* (this introduces a technical sense of 'modify', of course). The following principle is supposed, therefore:⁴⁸

- PA5 for any xs , for any ys , for all u , for all t , for all t'
 if t' is later than t , and each of the xs is distinct from each of the ys ,
 then if
 (1) the xs + the ys underlie u at t ,
 (2) the xs underlie u at t' , and
 (3) the xs are not modified between t and t' ,
 then the xs underlie u at t .

Suppose, for example, a heart has n cells as ^{elements} \forall at t . If it would survive the destruction (without replacement) of any one of those cells between t and t' , without its other cells being modified between t and t' , then according to PA5 the heart is underlain already at t by any $n - 1$ of its cells.

I shall not enter a discussion of various ways in which PA5 might be altered or refined (an refinement which immediately suggests itself, for example, would be to replace (3) by a more flexible condition). Note that already as it stands, the principle has the advantage of appealing to features (of individuals) about which we are amply informed by experience, and which are at least to some significant degree susceptible to empirical investigation.

For example, it is known that trees, generally speaking, can survive a loss of several of their principal branches. On the assumption that principal branches are among the elements of a tree (perhaps minor branches are elements of principal branches, and thus proper pre-elements of the tree), it may be concluded directly, by means of PA5, that if the xs are all the tree's elements except for one main

⁴⁸ the expression 'the xs + the ys ' is used in formulating PA5 in the following sense: the xs + the ys underlie u iff for some zs , each of the zs is either one of the xs or one of the ys , and each of the xs and each of the ys is one of the zs , and the zs underlie u .

branch, then the *xs* underlie the tree. And were we not sufficiently informed about this by experience, a variety of scientific procedures suggest themselves which would supplement our empirical knowledge appropriately.

In conjunction with the detailed study of the way the notion of a Unity can be applied in accounting for the nature of individuals, it is to be expected that generalizations will be formed regarding fundamental constraints on conditions under which individuals may be assumed to underlie a Unity. Such generalizations are developed on the basis of our understanding of the sortal concepts under which individuals of various types fall, together with scientific experience regarding such individuals. These constraints can then be formulated as postulates of a more developed theory.

I shall now give examples of several such general constraints which suggest themselves in the course of initial experimentation with the conception of individuals as Unities. It has to be emphasized that the list that follows has a tentative status, and is subject to all manner of revision on further consideration. It is chiefly intended to suggest the *type* of constraints that one might want to integrate into the theory of Unities.

It seems reasonable to assume, for example, that if each of two distinct collections of cells embodies a heart, then those collections overlap one another: there are cells which are members of both collections (the collection of shared elements may be called the 'intersection' of the two collections). We have, then,

Weak Sharing

for any *xs*, for any *ys*, for all *u*,

the *xs* underlie *u* and the *ys* underlie *u* only if for some *zs*, the *zs* are both among the *xs* and among the *ys*

A stronger assumption would be, that the intersection of any two embodying collections is itself an embodying collection. This could be formulated as follows:

Strong Sharing

for any *xs*, for any *ys*, for all *u*,

the *xs* underlie *u* and the *ys* underlie *u* only if for some *zs*, the *zs* are both among the *xs* and among the *ys*, and the *zs* underlie *u*

Suppose a certain castle is a Unity, underlain by several elements, among which are the castle's north wing and its south wing. Suppose the two wings do not overlap one another. Suppose the castle would survive if every one of its elements except one of the wings were destroyed. Then on PA5 either of the wings underlies the castle. If one wishes to maintain these assumptions about the survival of the castle, then one will need to reject both Sharing principles. Alternatively, one might accept either of these principles, and revise accordingly one's assumptions regarding the castle.

One may proceed to introduce constraints which involve not only the relations between any two embodying collections, but relations between many or all embodying collections. First, it seems plausible to suggest that there is a core collection of elements which is a part of any embodying collection of the Unity:

Weak Minimal Collection

for all u , if u is a Unity, then for some xs ,
 (for any ys , if the ys underlie u , then the xs are among the ys)

Moreover, it might be suggested that such a core collection would itself be an embodying collection:

Strong Minimal Collection

for all u , if u is a Unity, then for some xs ,
 (for any ys , if the ys underlie u , then the xs are among the ys),
 and the xs underlie u

It is tempting to identify the notion of a minimal embodying collection with the notion of a collection of *essential* parts. An essential part is a part on whose existence (and on whose appropriate relations to other parts) the existence of the whole depends. Clearly if a whole has essential parts, and the whole is a Unity, then the essential parts must either be members of any embodying collection, or they must be pre-elements of members of any embodying collection.⁴⁹

⁴⁹ More sophisticated versions of Strong Minimal Collection could be tailored to distinguish between different conceptions of the dependence involved between the whole and the parts. A human organism must, it would seem, have *a* heart, but *which* heart perhaps does not matter to the organism's identity (see Simons 1987, 294ff. on the distinction between generic and rigid dependence). Such distinctions would involve

Complementing constraints regarding minimal embodying collections, we have a constraint regarding a maximal embodying collection. Assumption A27 above implies the following principle:

Strong Maximal Collection

for all u , if u is a Unity, then for some xs ,
(for any ys , if the ys underlie u then the ys are among the xs)
and the xs underlie u

The observation that boundaries between individuals are often sharp suggests that the following principle has some plausibility:

Non-overlap of Unities

for all u , for all v , if u is a Unity and v is a Unity,
and $u \neq v$, then for all x , x is not an element of both u and v

It is perhaps not out of place to note that a variety of constraining principles follow from the basic assumptions A1-A27 and D1-D18. For example, Weak Supplementation has been counted among the basic mereological principles with which the theory of Unities complies. In a comment regarding assumption A11, doubts were raised as to whether this principle should indeed be asserted. A consequence of this assumption, is the following somewhat artificial, though not intolerable, constraint:

Plurality of Underliers

for all u , for any xs ,
if the xs underlie u , then the xs are at least 2 in number

Another example is the following principle, which is a consequence of the intransitivity of *is an element of*, and which suggests a hierarchical mereological conception of individuals:

Separation of levels

for all x , for all y , if x is an element of y , then x and y cannot be elements of one and the same Unity

making explicit the temporal modification of 'underlie', and specifying constraints involving relations between collections which embody the Unity at different times.

A great wealth of additional constraints suggest themselves in connection with processes involving flux of parts of individuals. Such constraints may state, among other things, conditions for survival of Unities in the face of mereological variation. For example, the following seems to be a plausible principle:

Dispensable Elements

for all u , for all x , for all t

if x is one of the elements of u at t

and x is not an element of u immediately after t

then u exists immediately after t only if x is not a member of *every* embodying collection of u at t

The latter principle may be developed into a necessary condition for survival over time through a series of mereological changes, as follows:

Continuity

for any xs , for all u , for any ys , for all v , for all t and t'

if the xs underlie u at t , and t is earlier than t' , and the ys underlie v at t' ,

then u is identical to v only if

for all z , if z is one of the xs but not one of the ys , then

z had ceased to be an element of u under circumstances complying with the principle of Dispensable Elements

Just as much as principles may be sought generalizing conditions under which loss of parts may occur, so we should look into principles associated with *gain* of parts. I leave the elaboration of such principles, as well as many others which suggest themselves on reflection, to another opportunity. What has been said should suffice to indicate how one might approach an explanatory programme utilizing the notion of a Unity.

9.2.2 *Dissolution of Problems faced by the Conception of Wholes as Neoclassical Sums*

In Chapter 6 we looked at some fundamental difficulties that are involved with some of the more important types of neoclassical sum. A brief consideration suffices to show that these difficulties do not burden the theory of Unities. At least in respect of these important difficulties, the theory of Unities shows itself therefore to be better suited to account for the way in^{which} the existence of a whole depends on its parts fulfilling certain conditionsⁱ (i.e having certain properties and relations to one another), for the way in which different wholes can be superposed with one another, and for the way in which wholes are capable of surviving the loss and gain of parts.

With regard to conditioned sums the point was made that the assumption that the whole is a sum forced upon the theorist the consequence that the conditions - defined with reference to the parts - under which the whole is assumed to exist are extraordinarily, and implausibly, sensitive. The juxtaposition of a new molecule to some of the old ones, at the periphery of a human organism, was sufficient to disrupt the collective capacity of the countless trillions of old molecules to have a sum, or at least to have a human organism as a sum. The change in the condition of the old molecules must be viewed as insignificantly small (the change can perhaps be described as follows: some of the old molecules at the periphery of the organism now have the relational property *is juxtaposed to a molecule of such and such kind in such and such direction*, in virtue of their relation with the new molecule, which they did not have before). The change in their collective capacity to make up something is great. The predicament is obviously unsatisfactory.

The theory of Unities, by contrast, is consistent with assumptions under which this difficulty does not arise. If, for example, the old molecules are assumed to underlie the organism before the new molecule is added, this does not entail that after the addition of the new molecule the old molecules cannot continue to underlie the organism, even if it is assumed that the old molecules and the new one *taken together* now underlie the organism. Of course, it is admitted that the organism cannot be at the same time a sum of the former and a sum of the latter.

However, wholes are not fundamentally sums, and so the organism may be made up of the old molecules (by being underlain by them) without^{being} a sum of those molecules.

The notion of a non-unique sum, it was argued, is committed to the claim that distinct entities may be presently indiscernible, a claim^{which} appears very dubious on closer examination. The theory of Unities, by contrast, makes ample provision for entities to be superposed with one another without being presently indiscernible. In 8.2.5 it was pointed out that a Unity may be superposed with a distinct Unity, or with a collection. Whenever two^{entities} u and v are superposed, it was assumed that for some xs , where each of the xs is a Unity, both u and v are sums of the xs (see D17). Such superposition is fully consistent, however, with present discernibility. For u and v necessarily stand in different relations to the xs .

Consider for example the illustration in 8.2.4. A is a sum of $a_1 \dots a_6$, and B is a sum of $a_1 \dots a_6$. However, each of the entities, $a_1 \dots a_6$ bears to A , and *not* to B , the relation is an *element of*. Each of the entities $a_1 \dots a_6$ bear to B only the relation is a *pre-element of*. Moreover, A and B are further discernible in that b_1 and b_2 are elements of B but not of A . These are differences between A and B which do not concern properties or relations they have at times other than the present, or in worlds other than the actual one.

Finally, let us compare neoclassical mereology and the theory of Unities regarding mereological variation. It was shown in Section 6.3 that the neoclassical notion of a mereological varying sum runs into difficulties in connection with the process whereby a part is gained (similar problems arise in connection with the converse process). It transpired that if it is accepted that the whole must be *determinately* a whole throughout the process, then there are three options: the whole must either cease to exist during a transitional period, or it must be made up both of the old parts and of the old parts + new parts, or it must flip *instantaneously* from being made up of the old part to being made up of the old parts + new parts.

Under neoclassical assumptions, none of these options seemed tenable. Under Unity-theoretical assumptions, by contrast, one immediately notices that the second of the three options is acceptable. Suppose a car is conceived of as a Unity, and suppose it has five elements: the body and four wheels. This implies that the car is

made up of (interpreting 'made up of' as 'underlain by') the body and the four wheels. It is consistent, however, with the assumption^s that the car is at the same time made up of the body and *three* of the wheels. The fact that being underlain by the body and four wheels is compatible with being underlain, at the same time, by the body and three wheels, allows the Unity theorist to avoid the implausibilities of intermittent existence or instantaneous change, not to speak of outright contradiction, which are the main options of the neoclassical theorist.

Section 9.3

The Paradox of Tibbles

Noteworthy consequences of the switch to a conception of wholes as Unities, as regards fundamental cruxes of the metaphysics of concrete entities, are revealed by considering that famous creature of thought-experiment, Tibbles the cat. The case of Tibbles is a classical philosophical paradox, where reflection on the simplest of our everyday beliefs leads to a striking contradiction, the avoidance of which does not seem possible without departing more or less radically from common sense-views. It has therefore been possible, by means of an analysis of this or closely analogous examples, to argue for a surprising range of *prima facie* implausible positions, such as the four-dimensional^{is it} view of concrete entities, the sortal relativity of identity, mereological essentialism, or the view that concrete entities may coincide.⁵⁰ I shall examine in what follows the paradox of Tibbles (which I shall refer to simply as 'the Paradox' in what follows), indicating the way it can be resolved under the assumptions of the theory of Unities, comparing the result with others based on the conception of wholes as sums.

Tibbles is a cat which is sitting on the mat at time t . Later, at t' , it is again sitting on the mat. Only in the mean time it has lost its tail. The Paradox is derived by assuming that an entity called 'Tib' exists, which comprises all of Tibbles's^{bodily} parts at t , and pointing out that on the evidence available at t , Tibbles and Tib are distinct from one another, while on the evidence available at t' , they are identical to one another, thereby arriving at a contradiction.

Suppose we consider Tibbles, uncontroversially, to be composed by the following parts: Head, Neck, Torso, Leg1, Leg2, Leg3, Leg4, and Tail.⁵¹ The

⁵⁰ Mark Heller (1988) argues from this paradox that concrete entities are not continuants, but rather have temporal parts; Peter Geach uses a variant of the example to argue for the sortal relativity of identity (see Geach 1980, 215f.); Roderick Chisholm uses it to support his mereological essentialist view (see Chisholm 1979); Peter Simons, and E.J. Lowe, argue for the coincidence of concrete entities on the basis of the Tibbles example (see Simons 1987, 117-127; Lowe 1989, Chapter 6).

⁵¹ For the purposes of this explanation, viewing Tibbles as composed of any mutually disjoint parts would do just as well, provided there are at least two parts which are disjoint from Tail.

foregoing discussion allows us to distinguish between two different things that one might mean, by claiming to refer to an entity by means of 'Tib' in the context of the example. On the one hand, one might mean (a) that one is referring to *a whole composed by* Head, Neck, Torso, Leg1, Leg2, Leg3, Leg4. On the other hand, one might mean (b) that one is referring to *a whole which corresponds to* (or, equivalently, to *a whole which is made up of*) Head, Neck, Torso, Leg1, Leg2, Leg3, Leg4.

Usually this distinction is not pointed out, presumably because on the prevalent conception of wholes as sums the two descriptions are equivalent. On Unity-theoretical assumptions, however, a whole which corresponds to the *xs* is not necessarily a whole which is composed by the *xs* (recall that we deny assumption D4 of Subsection 8.2.1). Therefore, according to the theory of Unities, the descriptions in (a) and (b) are not equivalent.

To describe the Paradox, with these comments in mind, in terms that are not question begging, let us use four names, 'Tibbles', 'Tib', 'Tibbles*' and 'Tib*', explaining their references by means of associated descriptions (note that alternative readings of 'Tib' and 'Tib*' are provided). To facilitate the formulation of such descriptions, let us say that parts of a cat which do not overlap any part of a tail are 'bodily parts' of the cat:

Tibbles : the cat sitting on the mat at *t*

Tib : either (a) the whole composed^{at t} the bodily parts of Tibbles
or (b) the whole corresponding to the bodily parts of Tibbles

Tibbles* : the cat sitting on the mat at *t'*

Tib* : either (a) the whole composed of the bodily parts of Tibbles*
or (b) the whole which corresponds to the bodily parts Tibbles*

Following Simons, we may express the Paradox by means of the following *prima facie* compelling argument (we avoid, however, by means of our distinction between 'Tibbles' and 'Tibbles*', and between 'Tib' and 'Tib*', his rather confusing use of the modifiers 'at *t*' and 'at *t'*'): ⁵²

According to the conception of wholes as sums, the two readings of 'Tib' and 'Tib*', respectively, are equivalent. Therefore one assumes that Tibbles has parts

⁵² See Simons 1987, 119.

which Tib does not have, and that they have different shapes, weights, overall lengths, etc. If that is so, it follows that

(1) Tibbles \neq Tib

Since Tibbles* and Tib* seem not to be discernible in any respect, one assumes that

(2) Tibbles* = Tib*

Assuming Tibbles and Tib are continuants, we have good reason to believe (in the context of the fictional example) that

(3) Tibbles = Tibbles*

and

(4) Tib = Tib*

Finally, since Tibbles $\stackrel{\text{by (3)}}{=}$ Tibbles* $\stackrel{\text{by (2)}}{=}$ Tib* $\stackrel{\text{by (4)}}{=}$ Tib, assuming the transitivity of identity we conclude

(5) Tibbles = Tib

And (5) obviously contradicts (1).

As Simons points out, conclusion (5) might be blocked by denying any one of several intuitively appealing theses: by denying that concrete macroscopic entities exist; by denying that they are continuants; by denying that they may survive the gain or loss of parts; by denying the absoluteness of identity; or by denying the transitivity of identity.⁵³ I shall not discuss these responses here. The theory of Unities allows us to avoid each of these denials, and I would like to concentrate on a comparison with other responses which have this feature in common with the theory of Unities.

If one accepts the conception of wholes as sums, and yet wishes to avoid all of the denials mentioned in the former paragraph, one seems to have two basic options. The first is to claim that arbitrarily prescribed parts of concrete entities, Tib being an example, cannot be assumed to exist. This allows one to deny (4). This is

⁵³ Regarding the last four response^s, see references in Simons 1987, 120. Regarding the first, see, e.g. Unger 1979 and 1981.

van Inwagen's position, at least as expressed in 'The Doctrine of Arbitrary Undetached Parts'.⁵⁴ Let us call this option the 'No-Tib' view.

The alternative option is to claim that distinct concrete entities may coincide, or at least may be in the same place at the same time. This allows one to claim that Tibbles* coincides with Tib*, even though they are not identical to one another, namely, to deny (2). This is the position of Wiggins, Doepke, Simons and Lowe, among others. Let us call this option the 'Many-Tibbles' view.

Before examining the two views individually, it should be emphasised that because they both embrace the conception of wholes as sums, they both face the substantial difficulties confronting that conception especially with regard to mereological variation (and in the case of the Many-Tibbles view, also with regard to non-uniqueness of sums), difficulties which were discussed in Chapter 6. As we shall see, however, the accounts offered of the Paradox according to these views lead (respectively) to additional problems and implausibilities, which this paradox helps to point out.

9.3.1 *The No-Tib View*

According to the No-Tib view, no entity exists which we would refer to by means of 'Tib', a term whose denotation we attempt to specify as noted above by one of the two descriptions (a) or (b). However, intuitively speaking, we do not seem to have any difficulty in conceiving of, or imagining, an entity thus described. In fact, on the basis of either description, we tend to imagine an entity which is very similar in many respects to Tibbles. It weighs almost as much as Tibbles; it is almost as long; it has, just like Tibbles, four legs, a head, abdomen, etc.; it has a fur of precisely the same colour and texture as that of Tibbles, etc. Indeed, on the face of it, Tib as we imagine it is in every way similar to a tailless cat, except for the fact that it is itself a proper part of a cat, whereas tailless cats cannot be proper parts of cats any more than cats with tails can.

⁵⁴ Van Inwagen 1981.

The intuition supporting such a conception of Tib seems to be closely associated with the widespread use of sortal terms which apparently apply to entities which are (typically) undetached parts of other entities, such as 'hand', 'head', 'toe', 'branch', 'knob' (as in 'the knob of the handle'), 'blade' (as in 'the blade of the sword'). It is true that such part-terms, as we may call them, are usually associated with relatively small parts of the respective wholes (a head, for example, is usually a relatively small part of the animal to which it belongs). But this is not always the case. The sword's blade may well be almost as large as the sword itself. Or, to take another example, the part of a sprawling building which we may designate by means of the term 'main building', may well be far larger than all the subsidiary wings of the building taken together.

Thus far speaking intuitively. An adequate^{view} of why we have this intuition (which we may call 'the Tib-intuition') would seem to be required of any acceptable account of the Paradox. If it is denied that we have this intuition, even an apparent paradox cannot be presented. If Tib were conceived of from the start, *prima facie*, as being of a completely different character to Tibbles, then we would not even be *inclined* to assent to (4). The No-Tib theorist, however, is not denying that the Paradox seems paradoxical.

The No-Tib view does not deny, then, that we have the Tib-intuition, but it rejects it as being mistaken. Moreover, it rejects it radically. Not only were we not referring to a tailless cat, when we used 'Tib'. We were not even referring to any single entity. The No-Tib view is therefore strikingly counter intuitive in this respect.

It may well be that any solution to the Paradox is bound to be counter intuitive in one respect or another. However, it is a particular drawback of a solution if while denying a powerful intuition it does not provide a satisfactory explanation as to why we might have such an intuition. If we are not shown how it comes about that a certain intuition of ours commits a mistake, we are likely to continue to follow that intuition. The No-Tib view does not provide any such explanation, and it fails to assuage the intuition that it *rejects*.

It might be suggested that we have the intuition because we use a term which makes apparent reference to a single entity. Perhaps a coherent account might be given of claims that we make apparently about a single entity, by showing how

such claims can be translated (equivalently) into claims about many entities. Simons claims that van Inwagen's claim that such an account is available has not been demonstrated. In his later book, *Material Beings*, however, van Inwagen does work out such an account in some detail, an account which I shall not attempt to assess here.⁵⁵

However, granting that such an account is available, it does not explain why such use of a term making only apparent reference to a single entity should engender, or be associated with, the (misguided) intuition that one is indeed referring to a single entity. The term 'flock' would seem to be an example of a term making apparent reference to a single entity (it is, like 'Tib', grammatically singular). However, it can hardly be claimed that use of this term is associated with the compelling intuition that a flock is a single entity.⁵⁶

Apart from its sharp conflict with the Tib-intuition, and apart from the problems it shares with any view based on the conception of wholes as sums with regard to accounting for mereological change, the No-Tib view gives rise to several other problems.

First we note that the view involves the assumption that there are *some* undetached parts. Plausible candidates for undetached parts of Tibbles which *do* exist, according to van Inwagen, are whole molecules, whole cells, and whole organs - such as the heart. Such parts of course themselves have parts, in turn, parts which are (by transitivity of *is a part of*) also parts of Tibbles.⁵⁷

⁵⁵ See Simons 1987, 121 (n.40); van Inwagen 1990.

⁵⁶ Whether a flock is, indeed, a single entity, is a different question (I have discussed issues associated with this question in Part I). It is quite possible that even though we do not have a compelling intuition that a flock is a single entity, it is, as a matter of fact, a single entity.

⁵⁷ It might perhaps be suggested that Tibbles has no parts at all, that is, no *actually* existing parts. This extreme version of the No-Tib view, which contrasts with van Inwagen's more moderate version, faces great difficulties, however. Suppose, for example, Jane has two hands, Left-Hand and Right-Hand, and suppose by saying this we do not imply that 'Left-Hand' and 'Right-Hand' designate actually existing undetached parts of Jane. Suppose SJ, SL and SR are regions of space which, on a common sense view, we would take to be completely occupied at *t* by Jane, Left-Hand and Right-Hand respectively (we ignore the indeterminacy of the boundaries between the supposed hands and the supposed arms).

On any view, SJ is (actually) occupied at *t* by something. It is hard to see how it could be maintained that a region of space can be completely occupied by a concrete entity, and yet none of the proper parts of that region are completely occupied by anything. For those parts of the region would then presumably be (actually) empty, which they are not. In particular, it would seem to follow that SL and SR are (actually) completely occupied by something at *t*. According to the extreme view we are considering, neither region could be

Let us grant the No-Tib view that Tib does not exist, but let us assume that more or less straightforwardly separable (in the surgical sense) organs of Tibbles do exist. Thus suppose the No-Tib theorist accepts the existence of entities such as Tibbles's heart, liver, pancreas, and brain. Now, suppose further that Tibbles's brain, which we may call 'Brain', may survive in separation from the rest of Tibbles's body, if appropriately sustained by artificial means - perhaps in some futuristic vat. Finally, suppose that in such circumstances - where we might imagine the rest of Tibbles's body to be destroyed - Tibbles survives. We may now reformulate the Paradox, replacing 'Tib' with 'Brain' and 'Tib*' with 'Brain*'. In this case, *ex hypothesi*, the No-Tib theorist accepts the existence of Brain. So stage (4) of the Paradox argument cannot be denied, and the contradiction is incurred.

Of course the No-Tib theorist might deny also that organs such as Brain exist (though the more types of parts of Tibbles are ruled out, the more difficult it would be to justify that Tibbles itself exist, for one would have to point out pertinent differences between those parts and Tibbles itself, which would justify the claim that Tibbles exists but not those parts). However, there seems to be no reason in principle why with respect to some types of parts of some entities an situation of the sort involving Tibbles and Brain should not obtain. And if it can obtain, in principle, then cases analogous to the Paradox can in principle occur, in connection with which the No-Tib view cannot avoid contradiction. This point alone would seem to be enough to show that the No-Tib view is untenable as a response to the Paradox.

occupied by a single entity which is a proper part of Jane, for no such proper (undetached) parts (actually) exists. Nor could each of the regions be occupied by many proper parts of Jane, for the same region.

It seems that the only remaining option is that Jane herself occupies both SL and SR. That is, the whole of Jane - remember, she has no proper parts - would occupy at one and the same time two disjoint regions of space. On this view, then, concrete entities are *repeatables*, in the sense in which universals are sometimes said to be repeatables (according to a familiar realist conception of universals associated with David Armstrong). Some may find this consequence not to be devoid of interest. Most will probably find it too bizarre to be taken seriously.

In any case, however, this extreme version of the No-Tib view cannot be a very tempting view in an age in which it is a common place of highly regarded science that a man is in some way or another made up of countless atomic particles, where there is no inclination to believe that such particles cease to exist once a man is "made up" of them.

The No-Tib view involves a further difficulty, however. There is much appeal in the suggestion that, crudely put, the existence of a whole can only be affected by what happens inside it, and not by what happens outside it (unless, of course, what happens outside it affects what happens inside it). To express this suggestion more precisely, let us use expressions of the form 'a change in the *xs*' as short for 'a change in the properties of, or relations between, the *xs*'. We may then formulate the following principle, which we call 'Autarchy of Wholes':

Autarchy of Wholes (I)

for any *xs*, for any *ys*, for all *t*,

if there is a whole which corresponds to the *xs* at *t*, and if the *xs* are among the *ys*, then

a change (commencing at *t*) in the *ys* does not cause there to be no whole which corresponds to the *xs*, unless the change in the *ys* either causes or partially consists in a change in the *xs*.

To illustrate the principle, suppose *a* is a table, a whole which corresponds to pieces of wood, the *bs*. Suppose *c* is a thin wooden board which has precisely the same surface shape and size as the table top. We do not think that by gluing *c* to *a* we have brought it about that there is no whole which corresponds to the *bs*. Indeed, we might suppose that that which is now (after the gluing) a whole which corresponds to the *bs* is not a table. We might claim, for instance, that it is a proper part of a table, and that a proper part of a table is not itself a table. And we might claim that after the gluing, *a* is no longer a whole which corresponds to the *bs*, but rather a whole which corresponds to the *bs* and *c*. These claims, however, are all compatible with there being still some whole which corresponds to the *bs* after the gluing.

(I)

The No-Tib view violates Autarchy of Wholes.[✓] It is easiest to see this if we consider the process involved in the Paradox of Tibbles in reverse. Suppose Tibbles has no tail, having lost it a short time ago. The tail is now surgically restored to Tibbles. Taking the conditions to be those of an ideal thought experiment, we assume that the process of re-attaching the tail neither involves nor causes any change in the parts of Tibbles (none of which overlap a tail), the *ps*. Suppose the *qs* are the parts of Tibbles and the tail. Before the operation, there is a whole which

corresponds to the *ps*, namely Tibbles. After the operation, the No-Tib view takes it, there is no whole which corresponds to the *ps*. So there were changes in the *qs*, which caused there to be no whole which corresponds to the *ps*, and yet there were no changes in the *ps* - in violation of Autarchy of Wholes. Therefore, if one embraces that principle, then one has a further reason for rejecting the No-Tib view.

9.3.2 *The Many-Tibbles View*

According to the Many-Tibbles view, Tibbles* is not identical to Tib*. The possibility of denying their identity to one another, notwithstanding the fact that they occupy the same region of space at the same time, is afforded by the neoclassical mereological denial of uniqueness of sums. Tibbles* and Tib* are both taken to be sums of the same parts, and yet, since sums need not be unique, it may be assumed that these sums are distinct from one another. Two versions of the Many-Tibbles view seem possible. On the Many-Tibbles₁ view, Tib (and thus also Tib*, given assumption (4) of the Paradox) is an entity of a fundamentally different kind to Tibbles. Tibbles is a cat, an entity susceptible to flux of parts. Tib is not a cat - it is mereologically rigid or invariable.

E. J. Lowe, who is clear about insisting on the Many-Tibbles₁ view, distinguishes between mereologically varying entities, which he calls 'integrates', and mereologically unvarying (and indeed invariable) entities, which he calls 'summative objects'.⁵⁸ Summative objects fall into two basic types, for which Lowe uses the terms 'collective' and 'aggregate' respectively. Collectives are conceived of as classical mereological sums of their parts. They may exist equally when their parts are closely packed together, as when the parts are scattered. The conditions of identity of aggregates, by contrast, involve the requirement that the parts of the entity in some sense adhere to, or are in contact with one another. Lowe assumes that Tibbles is an integrate, while Tib is an aggregate.

⁵⁸ See Lowe 1989, 89.

According to the second version of the Many-Tibbles view, Many-Tibbles₂, both Tibbles and Tib are entities susceptible to mereological flux. Such a position is suggested by David Wiggins's discussion of the need to distinguish between Tibbles and something he designates by means of the expression 'Tib + Tail'.⁵⁹ The former is distinguished from the latter not as being susceptible to mereological flux per se, but as being capable of surviving, in particular, the destruction of Tail. Tib+Tail, by contrast, according to identity conditions assumed in classical mereology with regard to sums, exists if and only if both Tib and Tail exists. However, since Wiggins takes both entities to be continuants (indeed, they typically coincide throughout the life of Tibbles), it would seem that both are assumed to admit of mereological change, and thus Tib, in particular, is assumed to admit of this. Simons, in his discussion of the example, is explicit that both Tib and Tail are subject to metabolism.⁶⁰

Both Many-Tibbles₁ and Many-Tibbles₂ are construed in keeping with the conception of wholes as sums, and as such are subject to the criticisms elaborated in Chapter 6. This applies not only to the account of mereological change, to which the No-Tib view was also committed, but also to the account of coincidence. According to both versions of Many-Tibbles, the two coincident entities at *t'* are indiscernible with respect to their present properties at *t'*, and thus they are saddled with the serious difficulties associated with this suggestion. Since No-Tib is not committed to coincident sums, it has an advantage in this respect over the Many-Tibbles views.

As with No-Tib, the accounts offered by the Many-Tibbles views of the Paradox lead both views into further implausibilities. As regards doing justice to what I have called the 'Tib-intuition', both views fare better than No-Tib. Indeed, Many-Tibbles₂ respects that intuition unqualifiedly. For Tib, on this view, is indeed similar to Tibbles in all the respects which the Tib-intuition takes it to be. Many-Tibbles₁, by contrast, can be seen as taking a middle position between the other two views. It accepts that there exists a single entity, to which we refer by means of the term 'Tib'. It rejects, however, that Tib is similar to Tibbles in respect of susceptibility to flux of parts.

⁵⁹ Wiggins 1979.

⁶⁰ See Simons 1987, 191.

The refusal of Many-Tibbles₁ to countenance flux of parts in Tib is not very happy - and has an arbitrary flavour. Going along with the basic story of the Paradox, if, after distinguishing between Tibbles and Tib we noticed that one of Tibbles whiskers has just been snipped off, we would be surprised at the suggestion that this resulted in the destruction of what we had taken to be Tib.

Moreover, the assumption that there is a category of concrete entities which are mereologically rigid, and yet which is, at the same time, emphatically distinguished from a classical mereological sum, seems very doubtful. After all, heaps of sand and lumps of clay - paradigmatic examples of Lowe's aggregates - *can* survive the loss of parts, at least if every day use of the terms 'heap' and 'lump' is to judge. I do not destroy a sand dune if I step in it and carry away a few grains which cling to my sandals, nor do I destroy a lump of clay by scraping a bit off with a knife. The assumption that there are such entities^(i.e. aggregates) cannot provide a solid foundation for a resolution of the Paradox of Tibbles.

Furthermore, a principle somewhat, but not substantially, less compelling than that of Autarchy of Wholes is violated by Many-Tibbles₁. At *t*, there is a whole which corresponds to the parts of Tib, which belongs to one ontological category (that of mereologically rigid entities). At *t'*, there is also a whole^{which} corresponds to the parts of Tib*, but this whole belongs to a category which is fundamentally different from that to which the whole which corresponds to those parts^{at t} belongs. Again, a principle might be formulated as follows:

Autarchy of Wholes (II)

for any *xs*, for any *ys*, for all *t*,

if there is a whole which corresponds to the *xs* at *t*, and if the *xs* are among the *ys*, then

a change (commencing at *t*) in the *ys* does not cause there to be a whole of a fundamentally different type, which corresponds to the *xs*, unless the change in the *ys* either causes or partially consists in a change in the *xs*.

To the extent that we embrace this principle, we are thus faced with an additional reason for rejecting Many-Tibbles₁.

The second version of Many-Tibbles is free from this complaint. It faces a problem, however, which renders it if anything even more unpalatable than the

first version. This can be seen by the simple means of envisioning a progressive series of loss of parts, each of which can be analysed in the manner presented in the Paradox.

Suppose the conditions at t are the same as those portrayed in the Paradox. At t' , however, instead of losing its whole tail, we assume that Tibbles loses *half* the length of its tail. Thus at t we have according to the view we are considering two mereologically non-rigid entities, one of which has a complete tail, whereas the other is half-tailed: Tibbles and Tib1 (where 'Tib1' is explained in a way analogous to that of 'Tib' above). At t' , both Tibbles and Tib1 are half-tailed, but even though they coincide, they are not identical.

Now suppose that by t'' (a time later than t'), Tibbles has lost the remainder of its tail. In that case, on the same assumptions, there is some entity Tib2 (again, 'Tib2' is explained in a similar way), distinct from Tibbles, with which Tibbles coincides at t'' . And what about Tib1? since we assume that it can survive the loss of parts, then we have no reason to deny that it too exists at t'' . And yet, the reasoning of the Paradox (on the assumptions of Many-Tibbles₂) compels us to acknowledge that Tib1 is not identical to Tib2, even though it coincides with it. And since we already know from considering the first episode of the process that Tib1 is distinct from Tibbles, we conclude that at t'' we have at least three coincident entities, Tibbles, Tib1 and Tib2.

Since on this view the loss of a tail does not *generate* an additional cat (or cat-like entity, at least) but only reveals the necessity to recognise its existence, the conclusion that at t'' there exist at least three coincident entities follows irrespective of the process whereby the tail was lost (that is, whether or not it was actually lost gradually, at two stages).

A similar consideration, however, would show that the proliferation of coincident entities does not stop here. Far from it. For in a similar way we could conclude, considering Tibbles to lose its tail in the course of a hundred stages, that at the end of the process there are one hundred entities coincident with the cat. Indeed, assuming Tibbles has lost its tail, the number of coincident entities that have to be acknowledged is no smaller than the number of ways in which a part could be removed from the tail. A rough calculation shows that (assuming that parts no smaller than single atoms can be separated from one another) the number

of entities which must be acknowledged as distinct from, but coincident with, the tailless Tibbles, is greater than the number of sand grains which would be needed to fill densely the volume of the known universe. It is hard to imagine that anyone could bring themselves to accept such a consequence in earnest.

In respect of ontological proliferation, then, we can distinguish between the three views, No-Tib, Many-Tibbles₁ and Many-Tibbles₂ as follows. Suppose Tibbles has lost its tail. According to No-Tib, there now exists Tibbles (as well as many proper parts of Tibbles - e.g. organs, cells, molecules, etc.). According to Many-Tibbles₁, there now exist two coincident entities Tibbles and Tib (as well as many proper parts of these). According to Many-Tibbles₂, there exist Tibbles, Tib1, Tib2, etc., i.e. innumerable coinciding entities. This is clearly a gradation of decreasing plausibility - other things being equal.

In respect of adequacy of account for the Tib-intuition, on the other hand, the views are related in an inverse gradation of implausibility. Many-Tibbles₂ justifies the intuition in full, Many-Tibbles₁ does so to a lesser degree, and No-Tib does so to the least degree.

9.3.3 *The Unity-theoretical View*

Looking again at the alternative possible explanations of the use of the term 'Tib' in describing the Paradox, we find that if reading (b) is accepted, the theory of Unities offers us an immediate and simple resolution. Taking Tibbles to be a Unity, the theory allows us consistently to assume that Tibbles is at the same time both a whole which corresponds to the parts of Tibbles which include the parts of the tail, and a whole which corresponds to the parts of Tibbles which do not include the parts of the tail. In that case, we simply deny step (1) of the argument of the Paradox, for we affirm that Tib ^{not} is distinct from Tibbles.

Note that this provides us with a fully satisfying explanation of the Tib-intuition, according to which both Tibbles and Tib can survive mereological change, and both are, in fact, cats. Conceiving of Tibbles as a Unity, we may consider both

Tib and Tibbles to be cats, indeed, we may consider them to be one and the same cat.

It is true that the Tib-intuition leads us to believe, *prima facie*, that Tib is a *tailless* cat, whereas Tibbles has a Tail. So it leads us to believe, *prima facie*, that Tibbles is distinct from Tib. The Unity-theoretical view accounts for this *prima facie* belief that they are distinct from one another, however, by presenting the relations between Tibbles and Tib to be analogous to the relations between the Morning Star and the Evening Star.⁶¹ In both cases, a *prima facie* belief that the entities involved are distinct from one another is grounded in the fact that they are referred to by means of different descriptions, descriptions which are compatible with, but do not imply that, the two entities are distinct from one another. The Morning Star is the star that appears in the morning, and the Evening Star is the star that appears in the evening. So they are *prima facie* distinct. But there is no contradiction in assuming that they are identical to one another.

Similarly, Tibbles is the whole which corresponds to parts which include parts of the tail, whereas Tib is the whole which corresponds to parts which do not include parts of the tail. Therefore, *prima facie* they are distinct from one another. However, the theory of Unities guarantees, by denying the assumption that wholes are fundamentally sums, that there is no contradiction in *asserting* nevertheless that Tibbles is identical to Tib.

In addition to fully satisfying the Tib-intuition (and in this respect being of comparable virtue to Many-Tibbles₂), it is remarkable that the Unity-theoretical view does not require the assumption that any coincident entities are involved in the story of Tibbles. Indeed, this view does not require us to acknowledge the existence of any entities other than those acknowledged by van Inwagen's minimalistic No-Tib view. In particular, we are relieved to avoid the radical implausibility of the consequence of Many-Tibbles₂, that there are innumerable mutually distinct entities which coincide with Tibbles after the loss of the tail.

But there is equally no need to presuppose the existence of doubtful mereologically rigid cat-like aggregates, which are required by Many-Tibbles₁. And even if we do believe that such aggregates exist, it is surely an advantage of the

⁶¹ See Frege 1892.

Unity-theoretical view that we are not compelled (at least on the grounds of considerations associated with the Paradox) to assume that they coincide with other, mereological varying entities. And furthermore, we are not committed to a violation of either version of the principle of Autarchy of Wholes, for we assume that one and the same whole corresponds to the non-tail-parts of Tibbles, both before and after the loss of the tail.

We have noted in the foregoing discussion that the conception of wholes as Unities is compatible with the assumption that concrete entities may occupy the same place at the same time. It may well be, for example, that a statue is a Unity underlain by Torso, Head, Arm1, Arm2, etc., while the lump of clay that constitutes it is a different Unity underlain by clay particles, which are pre-elements, but not elements, of the statue. However, the Unity-theoretical solution to the Paradox allows us to maintain that the phenomenon of coincidence of concrete entities is not instantiated in typical cases involving mereological change. The phenomenon may be viewed as being much less widespread than it is according to views of authors such as Wiggins, Simons, and Lowe.

Finally, and importantly, as noted in 9.2.2, the conception of wholes as Unities is exempt from the more general weaknesses associated with the account of mereological change, superposition, and conditionality of sums, as offered by theories which embrace the framework of the conception of wholes as sums.

Now, the resolution of the Paradox in terms of the theory of Unities, along the lines just noted, depends on the assumption that explanation (b) of the use of 'Tib', in the context of the argument of the Paradox, is acceptable. Reading (b) does seem to me to be highly plausible, as a component of the explication of the terms used in formulating the Paradox. And we recall that on the conception of wholes as sums, reading (b) is indeed equivalent to reading (a). However, let us suppose that it is insisted that reading (a) rather than (b) is the one which the propounder of the Paradox has in mind. It is still the case, in my view, that the resources of the theory of Unities allow us to provide a more plausible treatment of the Paradox than the competing No-Tib and Many-Tibbles views.

If we assume reading (a), then Tib is supposed to be a whole which does not overlap the tail. The theory of Unities recognises such a whole - namely, a collection of proper parts of Tibbles, none of which overlap the tail. Indeed, there may be

many such collections, which are superposed with one another (in the sense of 'superpose' explained in Subsection 8.3.5). Furthermore, such collections, I have argued, are ontologically innocent, for any collection construed in accordance with the assumptions laid down above is identical to its members (taken collectively).

Collections, since they are taken to comply with classical mereological principles, are assumed to survive the dispersal of their members. In this respect they are comparable to collectives, rather than to aggregates (which do not survive the dispersal of their parts), in Lowe's theory. It might be thought that the assumption that Tib is a collection, or a collective, does less justice to the Tib-intuition than the assumption that it is rather a mereologically rigid lump, or an aggregate. If that is so, it must be noted that the existence of such aggregates is consistent with the theory of Unities, so that if necessary, they might be pressed into service.⁶² Still, the Unity-theoretical view would be at an advantage over Many-Tibbles, since it is not subject to the general criticism[§] which apply to accounts of mereological change, coincidence, etc. which are based on the conception of wholes as sums.

A resolution of the Paradox along the lines proposed, assuming explanation (a) of the use of 'Tib', is by no means unsatisfactory, if compared to the resolutions offered by the other views. However, the resolution offered on the assumption that the use of 'Tib' is rather explained by (b) is much more attractive in its parsimoniousness and simplicity. Although reading (b) may sound less natural, I do think it faithfully expresses the observations which lead a common sense progression of thought into the Paradox of Tibbles. With all this in mind, reading (b), and the associated Unity-theoretical resolution of the Paradox, have the most to commend themselves among the positions which seem to be available.

⁶² A lump can be construed as a Unity which is embodied by a *unique* collection. Our assumptions are compatible with a Unity's being embodied by many distinct collections - this is one implication of the denial that wholes are fundamentally sums. However, they are equally compatible with a Unity's being embodied by a unique collection. If we wish to assume further that lumps have lumps as proper parts, we would have to reject one of the principles that have been tentatively offered in the sketch given above of ways in which the theory of Unities can be elaborated by introducing principles in addition to the essential principles constituting the theory of Unities. Namely, we would have to reject the proposed principle that no Unities may overlap one another. This principle is by no means required, however, in order to formulate the theory of Unities, so the prospect of its rejection gives no cause for worry.

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